LESSON 2.2

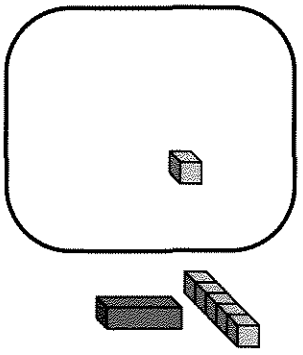
Adding and Subtracting

You will need:

the Lab Gear 

ASSOCIATIVE AND COMMUTATIVE LAWS

As you know, addition can be modeled with the Lab Gear by putting together collections of blocks on the workmat. For example, $x + 5$ means *put together x and 5* and $(x + 5) + -1$ means *put together x + 5 and -1*. This expression can be simplified by removing opposites, which would give us $x + 4$.

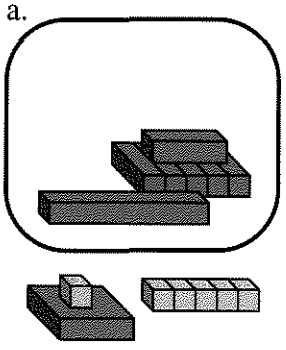


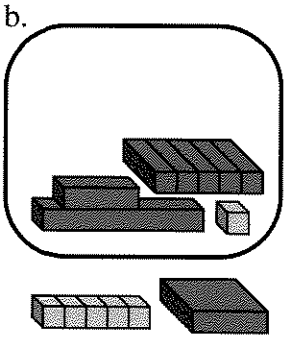
Note that the same figure could have been used to represent $x + (5 + -1)$. This is because, in an addition, quantities can be grouped in any way. This is called the *associative law for addition.*

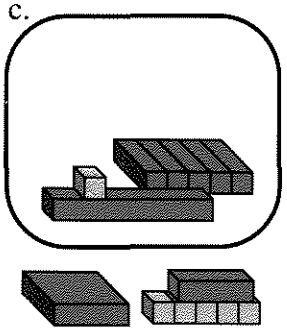
The same figure could have been used to represent $-1 + (x + 5)$, or $(5 + x) + -1$. This is because in an addition, you can change the order of the terms. This is called the *commutative law for addition.*

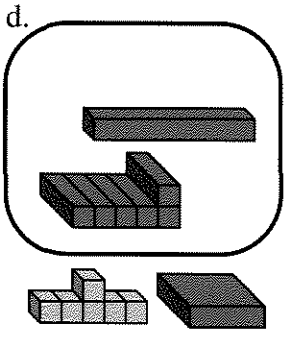
Finally, because of the commutative and associative properties, the -1 could have been shown upstairs on top of the x, or on top of the 5, instead of in the minus area. In every case, the expression would simplify to $x + 4$.

1. After simplifying these expressions, one will be different from the rest. Which one? Explain.

a. 

b. 

c. 

d. 

Add these polynomials. (In other words, remove opposites and combine like terms.) It may help to use the Lab Gear.

2. $(xy + 3x + 1) + (2x + 3)$

3. $(xy - 3x + 1) + (-2x - 3)$

4. $(xy + 3x - 1) + (-2x + 3)$

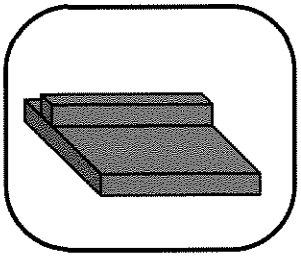
5. $(3 - 2x + xy) + (3x - 1)$

6. ◆ What do you notice about problems 4 and 5? Explain.

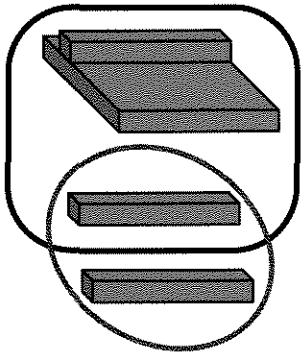
UPSTAIRS BLOCKS IN THE MINUS AREA

Here is a useful technique. To simplify upstairs blocks in the minus area, you can add zero, then remove opposites. For example, this figure shows how to simplify

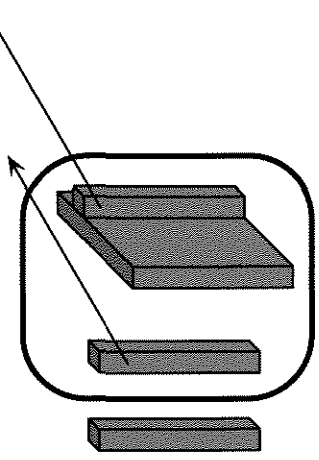
$$-(y^2 - y).$$



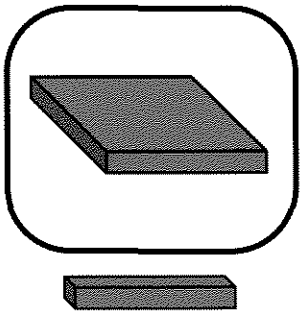
- **Add zero** by adding y inside and outside the minus area.



- **Remove opposites,** the matching blocks upstairs and downstairs.



- The simplified form is $-y^2 + y$. All the blocks are downstairs.



When working with the Lab Gear on the workmat, *simplifying* usually means

- removing opposites;
- combining like terms; and
- getting everything downstairs.

7. Model each expression using the Lab Gear. You will have to use both the minus area and upstairs blocks. Then simplify.

a. $-(5 - x)$ b. $-(x - 5)$

c. $3 - (x - 2)$ d. $(x - 2) - 3$

For problems 8–11 below:

- Build the first expression with the Lab Gear on the left side of the workmat.
- Next, compare each of the expressions a, b, c, and d to the original expression. (To make the comparison, build the expression on the right side of the workmat and simplify as needed.)

8. Which of these expressions are equivalent to $-(x + y)$?

a. $-x + (-y)$ b. $-x - y$

c. $-x + y$ d. $y - x$

9. Which of these expressions are equivalent to $-(x - y)$?

a. $-x + y$ b. $-x - y$

c. $-(y - x)$ d. $y - x$

10. Which of these expressions are equivalent to $-(y - x)$?

a. $x - y$ b. $-x + y$

c. $-y + x$ d. $-y - x$

11. Which of these expressions are equivalent to $-(-x + y)$?

a. $-x + y$ b. $-y - x$

c. $x - y$ d. $y - x$

12. **Generalization** For each expression below, write an equivalent one without parentheses. Do not use the Lab Gear.

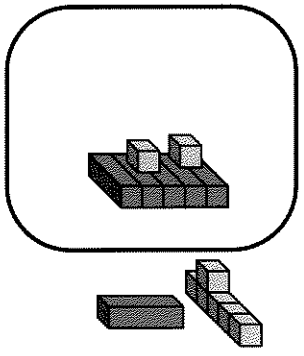
a. $-(a + b)$ b. $-(a - b)$

c. $-(-a + b)$

SUBTRACTION

The figure shows the subtraction

$$(x + 5 - 1) - (5x - 2).$$



13. Use what you learned in the previous section to simplify it.

14. Simplify, using the Lab Gear.

a. $x - (5x + 2)$ b. $x - (5x - 2)$

15. Simplify, with or without the Lab Gear.

a. $(6x + 2) - (3x + 1)$

b. $(3x - 2) - (6x + 1)$

c. $(6x - 1) - (3x - 2)$

d. $(3x - 2) - (6x - 1)$

16. In (a-c) find the missing expression. It may help to use the Lab Gear.

a. $-3x -$ _____ $= -4x$

b. $-3y -$ _____ $= -6y$

c. $-3y -$ _____ $= -2x - 4y$

17. **Summary**

a. Write a subtraction problem that you could model with the Lab Gear by putting blocks upstairs in the minus area.

b. Simplify this subtraction without using the Lab Gear. Explain the rule you are using.

18. How could you show the subtraction

$$y - -x$$

with the Lab Gear? (Hint: Remember about adding zero.) What would it look like after it is simplified? What is a rule you could use without the blocks to simplify this kind of expression?

19. Simplify without the blocks, $-(-a - b)$. Explain your answer.

REVIEW ***MINUS PUZZLE***

20. a. Using the Lab Gear, show -4 in five different ways.

b. What numbers of blocks can and cannot be used to show -4?