2.6 Finding Patterns

You will need:
graph paper

PARKING RATES

Two downtown parking garages charge different amounts, as shown by the following signs.

Ball Garage	
up to:	'U' pay:
1/2 hour	35 cents
1 hour	70 cents
1 1/2 hr	$1.05
2 hours	$1.40
3 hours	$2.65
4 hours	$3.90
5 hours	$5.15
6 hours	$6.40
7 hours	$7.65
all day	$8.90

Bear Garage	
up to:	fee:
1 hour	$1.05
2 hours	$2.10
3 hours	$3.15
4 hours	$4.20
5 hours	$5.25
6 hours	$6.30
all day	$7.25

1. If you park for two hours and five minutes, you have to pay the three-hour fee. How much is that at each garage?
2. People who work downtown tend to use one of the garages, and people who shop there tend to use the other. Explain why, with examples.
3. Lara notices that for the amount of time she is planning to park, the cost differen between the two garages is less than a quarter. How long is she planning to par
4. The parking fees at the Bear Garage mostly fit a pattern. Describe the pattern in words. Where does it break down?
5. The parking fees at the Ball Garage fit a more complicated pattern. Describe the pattern in words. Why might the owner Ball Garage have chosen a complicated pattern?

Analyzing numbers can be useful in making intelligent decisions. Here is an example.

6. Zalman owns an empty lot. He decides t convert it to a parking garage. He wants charge a fee that is not too expensive. H decides on these rules:
   - The fee should increase by a constant amount for each half-hour.
   - For parking times from a half-hour to nine hours, the fee should never be m than 25 cents higher than either Ball's Bear's fee.
   - The fee should be the highest possible fee that satisfies these rules.

   a. Explain why Zalman might have cho sen each rule.

   b. What rate should he choose? (For co venience in making change, it should a multiple of 5 cents.) Explain.
7. Graph the parking fees for all three garages. Put *time* on the horizontal axis, and *cost* on the vertical axis.

FIBONACCI SEQUENCES

The following numbers are called *Fibonacci numbers* after the Italian mathematician who first studied them:

1, 1, 2, 3, 5, 8, 13, 21...

8. Describe the pattern. Then give the next five Fibonacci numbers. (As a hint, if you have not yet discovered the pattern, look at the *Lucas numbers* — named after another mathematician — which follow the same principle: 1, 3, 4, 7, 11, 18, 29, 47, 76, 123...)

9. **Exploration** Look for patterns in the Fibonacci numbers. You may use addition, subtraction, or multiplication.

Definition: A *sequence* is an ordered list of numbers or expressions.

10. You can create your own Fibonacci-like sequence. Choose any two numbers, and use them as the starting values for a sequence like the ones described in problem 8. Name the sequence after yourself. Have a classmate check that your sequence is correct.

11. a. Find the first ten terms in a new sequence by adding the Fibonacci and the Lucas numbers. (The sequence should start: 2, 4, 6, 10, 16...) Is the resulting sequence a Fibonacci-like sequence? (Does it follow the same rule?)

b. Find the first ten terms in a new sequence by subtracting the Fibonacci numbers from the Lucas numbers. Compare your answer to the one in (a).

c. Find the first ten terms in a new sequence by dividing the sequence in (b) by 2. The result should be familiar.

12. Look for odd/even patterns in Fibonacci-like sequences including the original one, the Lucas sequence, and three named after students in your class. Explain.

13. Extend the Fibonacci and Lucas sequences to the left. In other words, what number should come before the first number? What number should come before that, and so on? Describe the resulting patterns.

MISSING NUMBERS

The following Fibonacci-like sequence fragments have numbers missing. Copy the sequences and fill in the blanks.

14. a. 0.5, 1.1, ___, ___, ___
b. 5, -4, ___, ___, ___
c. -6, -7, ___, ___, ___

15. a. ___, ___, ___, 11, 20
b. 2, ___, 7, ___, ___
c. ___, 3, ___, 9, ___

You may need to use trial and error for these.

16. a. 1, ___, ___, 11, ___
b. 12, ___, ___, 13, ___
c. ___, 8, ___, ___, 10

17. a. 1, ___, ___, ___, 11
b. 1, ___, ___, ___, 20
c. 2, ___, ___, ___, 19

18. a. 3, ___, ___, ___, ___, 29
b. 5, ___, ___, ___, ___, ___, 17

USING VARIABLES

19. Look at problem 17. Describe the relationship between the middle number and the outer numbers.
20. Create a five-term Fibonacci-like sequence in which the first two terms are x and y.
21. Check whether the pattern you noticed in problem 19 works for the sequence you just created. Explain.
22. Fill in the blanks for this Fibonacci-like sequence. -123, ___, ___, ___, 456
23. Extend the sequence you started in problem 20. Look for patterns.

FIBONACCI PUZZLE

24. How many Fibonacci-like sequences can you find that involve only positive whole numbers and include your age *in fourth place or later?* How about your teacher's age, or the age of a parent or adult friend

DISCOVERY ***PERIMETER ARRANGEMENTS***

25. **Exploration** Make sketches of some different ways that you could put together an x-block and an x^2-block in two dimensions. (They have to touch each other, but they don't have to make a rectangle.) Use your imagination. There are more than two arrangements possible. Is it possible to sketch all the arrangements you think up?
26. Find the perimeters of the arrangements you sketched in problem 19. Write each perimeter next to the sketch. Make sure you have found the largest and smallest perimeters possible.
27. Find two arrangements that have the same perimeter, but look as different from each other as possible.

REVIEW ***MISSING TERMS***

28. What terms are missing? More than one term may be missing in each problem.
    a. $3x^2 - 4x + \_\_\_ = -9x^2 + 8x + 7$
    b. $-x^2y + 6xy + \_\_\_ = 9x^2y + 8y$
    c. $3x^2 - 4x - (\_\_\_) = -9x^2 + 8x + 7$
    d. $-x^2y + 6xy - (\_\_\_) = 9x^2y + 8y$

PUZZLE ***MAGIC TRIANGLE***

29. Put an integer from -4 to 4 in each circle to get equal sums along each side of the triangle. Find as many different sums as you can.