LESSON 3.9

Equations and the Cover-Up Method

WRITING EQUATIONS

A seamstress makes dresses for a living. After an illness, she has only \$100 in her business bank account. She takes out a \$1000 loan at Algebank. The interest on the loan is \$15 per month if it gets paid back in the first year. She spends \$720 on dress-making materials, and keeps the rest in her bank account to cover additional costs, such as sewing machine repairs or whatever else may come up. Materials for one dress come to \$20. She makes two dresses a day, four days a week, and spends one day a week selling the dresses and dealing with other matters related to her business.

She sells as many dresses as she can to private customers for \$160 each, and the rest of the dresses to stores, for \$100 each. She needs \$750 a week for living expenses and puts any income over that in her bank account. She hopes to pay back her loan, and to make enough money so that when she needs to buy more materials, she does not have to take out another loan. Can the seamstress meet her goals? How could she improve her financial situation?

One way to think about a problem like this one is to break it down into smaller problems, and to write and solve equations for those. For example, let's write an expression that would tell us how much money the seamstress puts in her bank account every week.

1. **Exploration** Assume the seamstress has x private customers a week. Answer the following questions for one week, in terms of x.

a. How many dresses does she sell to stores?

b. How much money does she receive from private customers?

c. How much money does she receive from stores?

d. What is the total amount of money she receives every week?

e. How much of it is she able to put in her bank account? Simplify your answer.

If you answered the questions correctly, you should have ended up with the expression $60x + 50$ for the amount she deposits every week as a function of x. Let's say that she would like this amount to be \$300. This gives us the equation $60x + 50 = 300$. Remember that x is the number of private customers per week. We can now find out how many private customers she would need to deposit \$300 per week. All we need to do is solve the equation.

SOLVING EQUATIONS

Definition: Finding all the values of a variable that make an equation true is called *solving the equation.*

You have already solved equations by trial and error. The *cover-up method* is another technique for solving equations. It is based on the idea of working backwards.

Example 1: $60x + 50 = 300$

With your finger, cover up the term that has the x in it. The equation looks like

$$\square + 50 = 300.$$

Clearly, what's in the box is 250. So

$$60x = 250.$$

Think of a division that is related to this multiplication, and you will see that

$$x = 250/60$$

or $\mathbf{x = 4.1666....}$

So in order to deposit \$300 a week, the seamstress needs to have more than four private customers a week.

Example 2: This one is about a more complicated equation.

$$5 + \frac{3x - 1}{4} = 7$$

Cover up the expression $\frac{3x-1}{4}$. You get

$$5 + \square = 7.$$

Whatever is hidden must be equal to 2. So

$$\frac{3x - 1}{4} = 2$$

Now cover up $3x - 1$ with your finger.

$$\frac{\square}{4} = 2$$

What is under your finger must be 8. So

$$3x - 1 = 8.$$

Cover up the term containing x.

$$\square - 1 = 8$$

What's under your finger must equal 9. So

$$3x = 9$$

and $\mathbf{x = 3.}$

2. Check the solutions to examples 1 and 2 by substituting them in the original equations.

Solve each equation. Use the cover-up method, then check each answer by substituting.

3. a. $3(x - 10) = 15$
b. $3(x + 10) = 15$
c. $3 + \frac{x}{10} = 15$
d. $\frac{18}{x} + 12 = 15$

4. a. $34 - \frac{2x + 6}{2} = 4$
b. $34 - \frac{2x + 6}{2} = -4$

5. a. $21 = 12 + \frac{3x}{8}$ b. $12 = 21 + \frac{3x}{8}$

6. a. $5 + \frac{x}{6} = 17$ b. $5 + \frac{6}{x} = 17$
c. $5 - \frac{x}{6} = 17$ d. $5 - \frac{6}{x} = 17$

7. a. $3 = \frac{12}{x + 1}$ b. $3 = \frac{x + 1}{12}$
c. $3 = \frac{12}{x + 7}$ d. $3 = \frac{x + 7}{12}$

8. Make up an equation like the ones above that has as its solution
a. 4; b. -4; c. $\frac{1}{4}$.

Since the cover-up method is based on covering up the part of the equation that includes an x, it can be used only in equations like the ones above, where x appears only once. In other equations, for example

$$160x + 100(8 - x) - 750 = 300,$$

you cannot use the cover-up method, unless you simplify first.

9. Find out how many private customers the seamstress needs every week so that, at the end of four weeks, she has enough money in her bank account to pay back her loan and buy dress-making materials for the next four weeks. Use equations and the cover-up method if you can. Otherwise, use any other method. In either case, explain how you arrive at your answers.

REVIEW DIVIDING BY ZERO

10. Explain, using multiplication, why $20/5 = 4$.

11. Explain, using multiplication, why 20/0 is not defined. (Hint: Start by writing $20/0 = q$. Write a related multiplication. What must q be?)

12. Explain, using multiplication, why 0/0 is not defined. (Hint: Start by writing $0/0 = q$. Write a related multiplication. What must q be? Could it be something else?)

DISCOVERY BE PRODUCTIVE

Say that the product of a word is the product of the numbers corresponding to its letters. (For the letter values, see **Thinking/Writing 3.A**.) For example, the word `optic` has value

$$15 \cdot 16 \cdot 20 \cdot 9 \cdot 3 = 129{,}600$$

13. What is the product of the word `ALGEBRA`?

14. Find words whose product is as close to one million as possible.

15. 💡 Find words having these products. (Hint: It would help to find the prime factors of the numbers.)

a. 6	b. 8
c. 12	d. 14
e. 15	f. 16
g. 20	h. 24
i. 35	j. 455
k. 715	l. 2185
m. 106,029	n. 💡 4,410,000