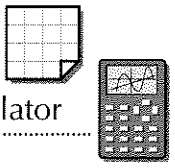
LESSON 4.5

Lines Through the Origin

You will need:

graph paper

graphing calculator (optional)



Definition: Since the graphs of first-degree equations are straight lines, these equations are also called *linear* equations.

1. Predict whether or not the graph of each linear equation will pass through the origin. Explain how you know, using graphs or calculations.
   a. $y = 4 - 2x$ b. $y = -2x$
   c. $y = 2x$ d. $y = 2x - 4$

2. Write two linear equations which you think will have graphs through the origin. Explain your reasoning.

RATIO

Lara and Lea were arguing about points and graphs. Lea said, "If the point (1, 4) lies on the line, then the point (2, 8) must also lie on the line." Lara showed her that she was wrong by drawing three lines.

3. On graph paper, draw a line that goes
   a. through both points;
   b. through (1, 4) but not through (2, 8);
   c. through (2, 8) but not through (1, 4).

4. Of the three lines you drew in problem 3, which goes through the origin?

5. a. Plot and label at least three more points that are on the line through (1, 4) and (2, 8).
   b. Find the equation of the line through (1, 4) and (2, 8).

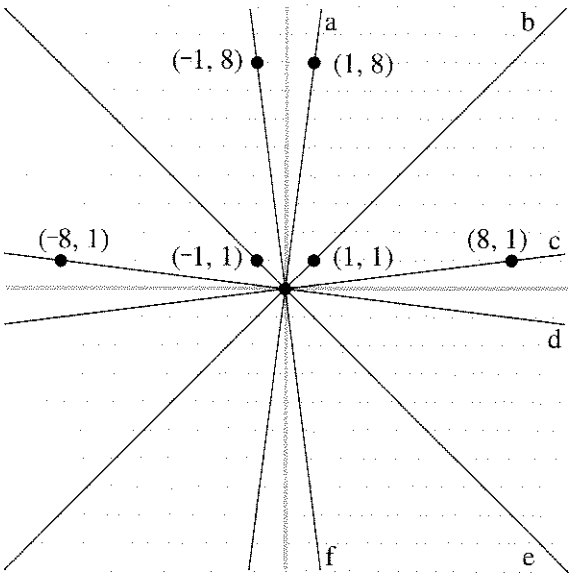
6. Plot these eight points on the same axes. Label them with their coordinates.
   (1, 2) (-1, -2) (1, -2)
   (-1, 2) (3, 6) (-3, -6)
   (6, 3) (6, -3)
   a. Draw a line connecting each point with the origin. Which points lie on the same line through the origin?
   b. Explain how to find the equations of the lines you drew.

Definition: The *ratio* of a to b is the result of the division a/b.

Example: The ratio of 6 to 3 is 6/3 or 2, while the ratio of 3 to 6 is 3/6, or 1/2, or 0.5.

7. a. Write two (x, y) pairs for which the ratio of y to x is 1/3.
   b. Plot these two points and graph the straight line through them. Find the equation of the line.
   c. Write two (x, y) pairs for which the ratio of y to x is 3.
   d. Plot these two points and graph the straight line through them. Find the equation of the line.

8. For each line in the graph below, find three points on the line. Then find an equation for the line.



9. Explain how you can find more points on the same line through the origin as (4, 5) without drawing a graph. Then check by graphing the line. Find the equation of the line.

Lea noticed that for the points (1, 4) and (2, 8) the ratio of the y-value to the x-value was the same. That is, $4/1 = 8/2$. She guessed that (100, 400) will lie on the same line through the origin because the ratio of the y-value to the x-value is also 4.

10. Tell whether or not you agree with Lea, and why.

11. Find a point whose coordinates have the same ratio of y to x as the point (4, 12). Does this point lie on the same line through the origin as (4, 12)? If so, find the equation of this line.

12. a. Graph the line through (-1, 2) and (3, 4).
b. Is the ratio of 5 to -10 equal to the ratio of -1 to 2?
c. Is the point (5, -10) on the line? Explain why or why not.

13. Generalization
a. What would be the ratio of the coordinates of points on the line through the origin and the point (a, b)? Explain.
b. If $b/a = d/c$, what can you say about the line joining (a, b) to (c, d)? Explain.

14. Summary Explain what ratio has to do with lines through the origin.

SPEED

The table shows the amount of time it took several people to travel the distances given.

Person	Time (hours)	Distance (kilometers)
A	3	80
B	7	140
C	12	320
D	1	30
E	2	30
F	1	20
G	5	150

15. a. Draw a pair of axes and label the vertical axis *distance* and the horizontal axis *time*. Plot and label the points in the table. Draw lines connecting each point with the origin.
b. Which points lie on the same line through the origin?

16. Use the table and your graph to answer these questions.
a. Which people are traveling at the same speed?
b. Who is traveling faster, *A* or *B*?
c. How far will *A* have traveled in four hours?

17. a. *H* has been traveling two hours at the same speed as *G*. Add *H* to your graph.

b. I have been traveling four hours at the same speed as *A*. Add me to your graph.

18. *J* is traveling faster than *B* but more slowly than *D*. Draw one possible distance-time graph showing *J's* progress.

19. Each line you drew has an equation that relates distance to time. Find these equations and add them to your graph.

20. Summary

a. Explain how one can think of speed as a ratio.

b. If you are given time and distance for two travelers, explain how to use calculations or graphs to compare their speeds.

DISCOVERY HAPPY NUMBERS

Take the number 23.
Square each digit and add.
$2^2 + 3^2 = 13$
Repeat this process.
$1^2 + 3^2 = 10$
$1^2 + 0^2 = 1$
$1^2 = 1$
The final result is 1.

Whenever the final result of this procedure is 1, the original number is called a *happy* number. So 23 is a happy number.

21. There are 17 two-digit happy numbers. Try to find all of them. It will save you time and help you look for patterns if you keep a neat record of the above process for each number.

22. Describe any patterns you notice.