LESSON 5.11

Averages and Sums

MEANS AND MEDIANS

You probably know how to find the average of a set of numbers. For example, the ages of the people in Tina's family are 10, 48, 20, 22, and 57. You would find the average age by adding the numbers and dividing by 5.

$$\frac{10 + 48 + 20 + 22 + 57}{5} = 31.4$$

We call this average the *mean*. Another kind of average is the *median*.

Definition: The *median* is the middle of a set of numbers that are in order from least to greatest. To find the median of an even number of numbers, find the two middle numbers and find their mean.

Examples: To find the median age in Tina's family, first write the numbers in ascending or descending order.

10 20 **22** 48 57

The median is **22.**

These are the ages of people in Lana's family: 52, 20, 15, and 53. To find the median, first write the numbers in ascending or descending order.

53 **52** **20** 15

Compute the mean of the middle two numbers: $\frac{52 + 20}{2} = \mathbf{36}$, so the median is 36.

1. Find the mean of the ages in Lana's family. Compare it with the median.

2. Make up a sequence of seven numbers in which

a. the mean is less than the median;

b. the median is less than the mean;

c. the mean and the median are equal.

3. Repeat problem 2 for a sequence of eight numbers.

4. **Exploration** Find some sequences of numbers in which the mean and the median are equal. Work with your classmates and compare your answers. What can you conclude about these sequences? Write a summary of your conclusions, including examples. (At least one example should be an arithmetic sequence, and at least one should not be.)

5. For each example below, make up two sequences that fit the given description.

a. The greatest term is 19, and both mean and median equal 10.

b. There are six terms. The greatest is 25, the mean is 10, and the median is less than 10.

c. There are seven terms. The least is -60, the median is 18, and the mean is less than 18.

d. The mean and the median are both -4. There are nine terms.

6. If possible, make up an arithmetic sequence that fits each description in problem 5. If it's not possible, explain why not.

MEANS AND SUMS

7. Find the mean and the sum of each arithmetic sequence.

a. -2, -14, -26, -38, -50, -62, -74

b. -5, -1.8, 1.4, 4.6, 7.8, 11, 14.2, 17.4

c. 31, 29, 27, 25, 23, 21

d. 17, 20, 23, 26, 29, 32

8. Study your answers to problem 7.

a. In which cases was the mean one of the terms in the sequence?

b. When the mean was not one of the terms in the sequence, how was it related to those terms?

c. How are the number of terms, the mean, and the sum related?

Suppose we wanted to find the sum and the mean of this arithmetic sequence:

3, 9, 15, 21, 27, 33, 39, 45, 51.

Using Gauss's method, write the sequence twice, once from left to right, and then from right to left.

3	9	15	21	27	33	39	45	51
51	45	39	33	27	21	15	9	3

9.

a. Add each column above.

b. Find the mean and the sum of the sequence.

c. How are your answers to (b) related to the sum of each column?

10. Using your results from problem 9, find a shortcut for calculating the sum and the mean of an arithmetic sequence. Try it on the examples in problem 7, comparing your results with your previous answers.

11. Find the sum and the mean of each arithmetic sequence described.

a. The sequence has 15 terms. The first term is 12, and the last term is 110.

b. The first term is -11, and the last term is -33. Each term is obtained by adding -2 to the previous term.

c. The first term is -14, and the difference between consecutive terms is 5. There are 41 terms in the sequence.

d. The first term is 7, and each term is obtained by adding -1.4 to the previous term. There are eight terms in the sequence.

12. Generalization Find the sum and the mean of each arithmetic sequence.

a. The first term is *b*, and the final term is 5. There are six terms in the sequence.

b. The first term is *b*, and the final term is *f*. There are 10 terms in the sequence.

c. The first term is *b*, and the final term is *f*. There are *n* terms in the sequence.

d. The first term is *b*, and each successive term is obtained by adding *d*. There are *n* terms in the sequence.

THEATER SEATS

Seats in a theater are arranged so that there are 35 seats in the front (first) row, 38 in the next row, 41 in the row behind that, and so on, adding three seats each time.

13. How many seats are in the

a. 10^{th} row? b. the n^{th} row?

14. How many total seats are needed if the theater has

a. 26 rows? b. *n* rows?

15. How would your answers to questions 13-14 be different if there were 34 seats in the first row?

16. Suppose there were 35 seats in the first row, 37 in the next, and so on, adding two seats each time. How would your answers to questions 13-14 be different?