LESSON 7.2

Square Windows

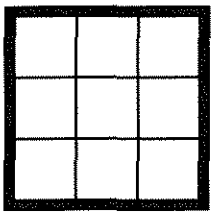
You will need:

graph paper

THREE TYPES OF PANES

The A.B. Glare window store has started selling a new kind of window. These windows can be made to order by combining three types of square window panes. Each pane measures one foot on each side. The three types of panes are shown below: corner panes, edge panes, and inside panes.

A 3-foot-by-3-foot window is shown below. It was made by putting together 4 corner panes, 4 edge panes, and 1 inside pane.



1. Sketch a 4-foot-by-5-foot rectangular window. How many panes of each type were used to make it?

SQUARE WINDOWS

2. **Exploration** An architect was asked to design a recreation hall. He was going to use the A.B. Glare window panes described above. The building code imposes a limit of 72 square feet for the total area of all windows in the main part of the hall. The architect decides to consider various combinations of square windows such that their total area is exactly 72. Find several such configurations, and for each one, find the total number of each type of pane the architect will need.

The architect is not the only one to like square windows. To save time when customers ask for them, Lara is assembling kits with the correct number of corner panes, edge panes, and inside panes to make square windows of various sizes.

3. Make a table to show how many panes of each type are needed for a 2-by-2 window, a 3-by-3 window, and so on, up to a 10-by-10 window.

4. Study the table from problem 3. Which increases the fastest: the number of corner, edge, or inside panes? Which increases the most slowly? Why?

5. Make three graphs of the data in your table, on the same set of axes.
   a. Graph the number of corner panes as a function of the length of the side of the window. For example, since a 3-by-3 window uses four corner panes, the point (3, 4) would be on your graph.
   b. Graph the number of edge panes as a function of the side length.
   c. Graph the number of inside panes as a function of the side length.

6. Study your graphs. Which is the steepest? Explain why.

7. Generalization

a. Write a formula for the number of panes of each type in an x-by-x window. Explain each formula in reference to a sketch of such a window.

b. How are the formulas related to the graphs in problem 5?

8. Add up the algebraic expressions for the numbers of each type of pane. If you did your work correctly, the sum should be very simple.

9. Find the number of corner, edge, and inside panes needed for a 100-by-100 window.

COMPARING SIZES

10. Lara has too many window kits of some types and not enough of other types. She has too many kits for 2-by-2 windows and not enough for 3-by-3 windows. How many panes of each type would she have to add to a 2-by-2 kit to convert it to a 3-by-3 kit?

11. Answer question 10 if Lara wanted to convert

a. a 5-by-5 kit to a 6-by-6 kit;

b. an 8-by-8 kit to a 9-by-9 kit.

12. Generalization How many panes of each type would Lara have to add if she wanted to convert an N-by-N-foot kit to an $N+1$-by-$N+1$-foot kit. Explain, using a sketch of an $N+1$-by-$N+1$ window.

13. How many panes of each type would Lara have to add if she wanted to convert an N-by-N-foot kit to an $N+M$-by-$N+M$-foot kit. Explain, using a sketch of an $N+M$-by-$N+M$ window.

MAKING THE MOST OF INVENTORY

14. Suppose you have 12 panes of each type in inventory.

a. What is the largest square window you could make? Give the size of the window and tell how many panes of each type you would have left over.

b. What is the largest square window you could make with the remaining panes? Continue until no more windows can be made. Give the size of all the windows and the number of each type of pane left at the end.

15. Repeat problem 14 for:

a. 20 panes of each type;

b. 100 panes of each type.

16. Now assume that instead of trying for the largest possible square window, you try to make any number of square windows, with the goal of having *as few panes as possible left over.*

a. If you start with 100 panes of each type, what size windows should you make? What will be left over?

b. Compare your answers with other students' answers.

PREVIEW *BIGGER WINDOWS*

17. Suppose each pane, regardless of the type, costs \$1.00.

a. Make a table and a graph showing the cost of the window as a function of the side length.

b. Al knows that an 8-by-8 window costs \$64.00. He thinks that a 16-by-16 window should cost twice as much, but he isn't sure. What do you think? Explain your opinion.

c. A 16-by-16 window costs how many times as much as an 8-by-8 window?