LESSON 7.4

Differences of Squares

You will need:

- the Lab Gear
- graph paper
- scissors

1. **Exploration** Which is greater, $3^2 - 2^2$ or $(3 - 2)^2$? Which is greater, $8^2 - 5^2$ or $(8 - 5)^2$? Is it ever true that

$$y^2 - x^2 = (y - x)^2 \text{ ?}$$

Is it ever true that

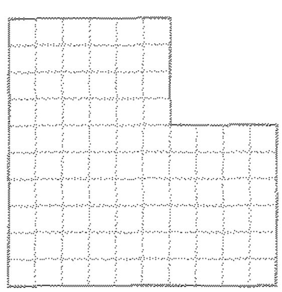
$$y^2 - x^2 < (y - x)^2 \text{ ?}$$

Experiment, and write a paragraph summarizing your work and your conclusions.

CUTTING A SQUARE OUT OF A SQUARE

Problems 2-4 show how to model the difference of two squares geometrically.

2. Cut a 10-by-10 square out of graph paper. Then, out of the corner of this square, cut a 4-by-4 square. The remaining paper should look like this.



3. The size of the remaining paper represents the *difference* of the 10-by-10 square and the 4-by-4 square. Its area is $10^2 - 4^2$ square centimeters. How many square centimeters is this?

4. The odd-shaped figure you have left after cutting out the 4-by-4 square can be rearranged into a rectangle. You can do this by making a single cut in the paper. Try it. Sketch the resulting rectangle and label its length and width.

5. **Generalization** Repeat problems 2-4 for some other differences of squares. (For example, try cutting a 3-by-3 square out of a 7-by-7 square. Try several others.) Can the resulting shape always be rearranged into a rectangle, no matter what two numbers you use? Can you use fractions? What are the dimensions of the rectangle? If it can always be arranged into a rectangle, explain why. If not, explain when it is possible and when it is not possible. Give examples, using sketches.

USING VARIABLES

Use the Lab Gear to do problems 6-9.

6. Trace the x^2-block on a piece of paper and cut out the square. Then trace a 1-by-1 square in the corner of the x^2-paper and cut it out. What difference is represented by the remaining paper?

7. Show how you can rearrange the remaining paper into a rectangle. Make a sketch showing the dimensions of the rectangle.

8. Repeat problems 6 and 7 for the following squares. You do not have to do the actual cutting unless you want to, but your sketches should be traced in the correct sizes.

a. Cut a square having area 4 out of a square having area x^2.

b. Cut a square having area x^2 out of a square having area y^2.

c. Cut a square having area 9 out of a square having area y^2.

d. Cut a square having area 25 out of a y-by-y square.

9. **Generalization** Make a sketch showing what remains after a square having area a^2 has been cut out of a square having area b^2. Then show by sketching how this can be rearranged into a rectangle. What are the dimensions of this rectangle?

FACTORING A DIFFERENCE OF SQUARES

When you cut a square out of a square, the area of the remaining paper is the *difference* of the two squares. When you rearrange this paper into a rectangle and write the area as *length* · *width,* you are writing this difference as a *product,* or factoring. Later in this course you will find this factoring technique helpful in solving equations.

10. Which of these is a difference of two squares?

a. $4x^2 - 16y^2$

b. $4x^2 + 16y^2$

c. $(x - y)(x - y)$

d. $(a - b)^2$

11. Write these differences as the product of two factors.

a. $x^2 - 9$ b. $y^2 - 25$

c. $25 - x^2$ d. $4x^2 - 16$

12. Factor.

a. $9y^2 - 25$ b. $9 - 25x^2$

c. $9y^2 - 25x^2$

13. **Generalization** In this lesson you found a technique for factoring a difference of two squares. However, in all the examples you have done, you have assumed that the first square was larger than the second. Does the pattern work if the first square is smaller than the second? That is, if a is less than b, is it still true that

$$a^2 - b^2 = (a - b)(a + b)?$$

Experiment, using some numbers, and explain your conclusions.

REVIEW *THE LAB GEAR MODEL*

14. Use the corner piece to multiply $(y + 5)(y - 5)$. Remember to simplify.

15. Show $y^2 - 25$ with the Lab Gear. Show how you can add zero and rearrange the blocks so that the uncovered part forms a rectangle. What are the dimensions of the rectangle?

16. Explain how one can use the Lab Gear to factor

a. $x^2 - 1$; b. $y^2 - x^2$.

17. **Summary** Write a paragraph summarizing what you learned in this lesson about differences of squares. Use sketches and examples.

18. Arrange Lab Gear blocks to show a square having area $(x + 5)^2$.

a. Using the blocks, remove a square having area x^2 out of the square having area $(x + 5)^2$, and rearrange the remaining blocks as a rectangle. Write its dimensions.

b. Repeat part (a) and remove a square having area 25.

c. What other squares can you remove from $(x + 5)^2$? Remove one, and rearrange the remaining blocks into a rectangle.

d. Explain how parts (a), (b), and (c) are examples of the pattern you learned about earlier in this lesson.

19. Write each difference as a product of two factors.

a. $(y + 4)^2 - y^2$

b. $(y + 4)^2 - (y + 3)^2$

c. $(y + 4)^2 - (y + 1)^2$

20. Factor. $(y + 2)^2 - (x + 5)^2$

REVIEW SOLVING EQUATIONS

Solve these equations using the cover-up method.

21. $\frac{5 - x}{7} = \frac{8}{14}$

22. $2 - \frac{x - 2}{3} = \frac{2}{3}$

23. $3 + \frac{2 + x}{5} = \frac{19}{5}$

24. $\frac{-7}{6} = \frac{x}{4}$

25. $6 - \frac{14}{x} = \frac{5}{2}$

26. $\frac{2 + x}{8} = \frac{5}{3}$

27. $\frac{1}{x} = 2$

28. $\frac{1 + x}{3} = \frac{2}{9}$

29. $\frac{4}{x} = 5$

30. $\frac{4}{x - 1} = 5$

31. $\frac{4}{3x - 1} = 5$

32. $\frac{4}{x + 4} = \frac{5}{6}$