LESSON 7.6

How Many Solutions?

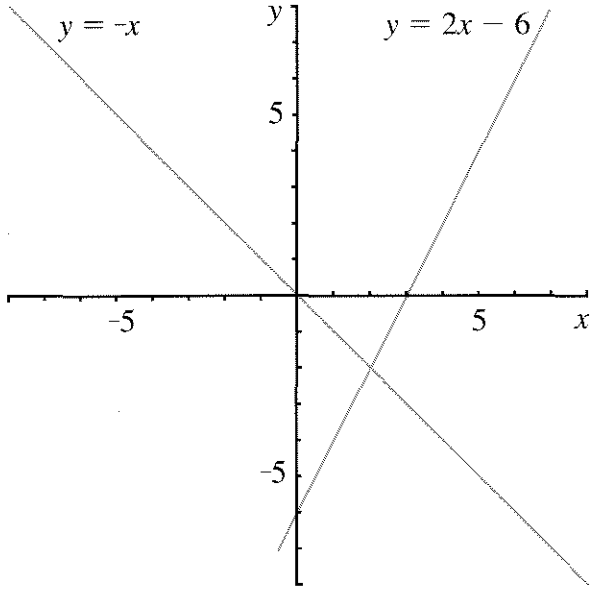
You will need:

graph paper

graphing calculator (optional)

LINEAR EQUATIONS

As you learned in Chapter 6, graphing is one way to find solutions to equations. For example, consider the equation $2x - 6 = -x$. This equation can be solved by graphing the lines $y = 2x - 6$ and $y = -x$ on the same axes.



1. a. From the graph above, estimate the point of intersection of the lines $y = 2x - 6$ and $y = -x$.

b. Use algebra to solve the equation $2x - 6 = -x$.

2. The linear equation $2x - 6 = 2x$ has *no solution.* Show that this is true by graphing the lines $y = 2x - 6$ and $y = 2x$. Explain how your graph shows that the equation has no solution.

3. Tell how many solutions each equation has. Use graphs if necessary.

a. $5x - 6 = 5x - 7$

b. $5x - 6 = 0.5(10x - 12)$

c. $5x - 6 = x$

4. For all the equations in problem 3 that have one solution, find the solution.

5.

a. Write and solve a linear equation that has only one solution.

b. Write a linear equation that has an infinite number of solutions.

c. Write a linear equation that has no solution.

6. Is it possible for a linear equation to have two solutions? Three solutions? Explain your answers, using graphs if possible.

QUADRATIC EQUATIONS

Definition: Second-degree equations are called *quadratic equations.*

Example: These are all quadratic equations.

$x^2 = 45$

$3x^2 - 15 = 6x + 2$

$6x^2 + 5x + 8 = 0$

You will learn several methods for solving quadratic equations. In this lesson, we will use graphing. Use a whole piece of graph paper for problems 7-11.

7. Draw a pair of axes on a full page of graph paper. Show all four quadrants. Graph $y = x^2$ very carefully.

8. On the same pair of axes, graph these lines and label them with their equations.

a. $y = 6x - 12$

b. $y = 6x - 9$

c. $y = 6x - 5$

9. Label the point or points of intersection of each line with the graph of $y = x^2$.

One of the lines you drew touches the graph of $y = x^2$ at only one point.

Definition: A line that touches a graph at only one point is *tangent* to the graph.

10. Which of the lines you drew is tangent to the graph of $y = x^2$?

11. Use the graphs to solve these equations.

a. $x^2 = 6x - 12$

b. $x^2 = 6x - 9$

c. $x^2 = 6x - 5$

HOW MANY INTERSECTIONS?

12. a. Draw a graph of $y = x^2$.

b. On the same axes, draw a line that does not intersect $y = x^2$. Write the equation of the line.

c. Repeat part (b) for another line that does not intersect $y = x^2$.

13. a. Draw a graph of $y = x^2$.

b. On the same axes, draw a line that intersects $y = x^2$ at only one point. Write the equation of the line and label the point of intersection.

c. Repeat part (b) for another line that intersects $y = x^2$ at only one point.

14. a. Draw a graph of $y = x^2$.

b. On the same axes, draw a line that intersects $y = x^2$ at two points. Write the equation of the line and label the points of intersection.

c. Repeat part (b) for another line that intersects $y = x^2$ at two points.

15. Refer to your answers to problems 12-14. Use them to write and solve a quadratic equation that has

a. one solution;

b. two solutions;

c. no solutions.

16. Use graphs to estimate the solutions to these equations.

a. $x^2 = -6x - 11$

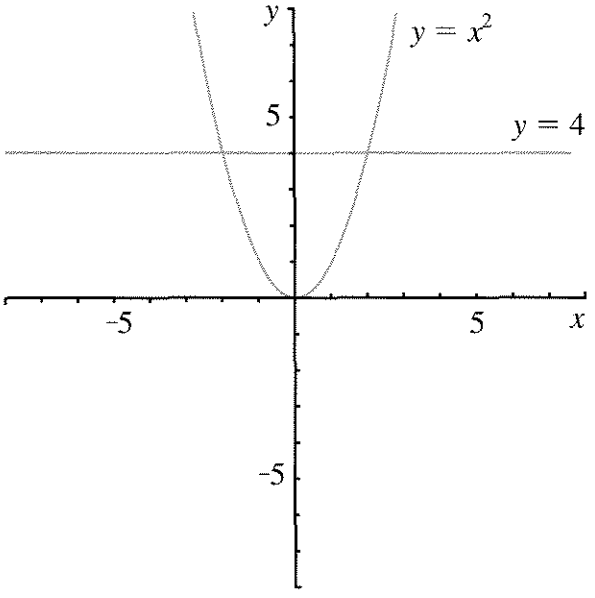
b. $x^2 = -6x + 11$

c. $-x^2 = 6x + 11$

17. Write the equation of a line that is tangent to $y = x^2$ at the point (-4, 16).

WHICH GRAPH SHOULD YOU USE?

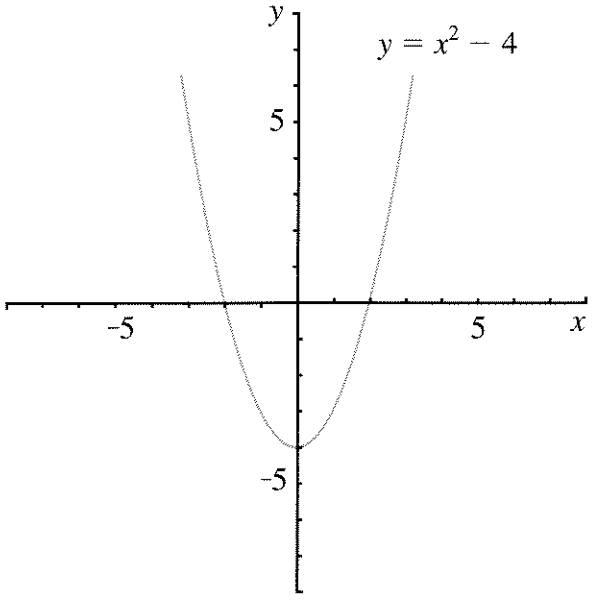
The solution of the equation $x^2 = 4$ can be found by graphing $y = x^2$ and $y = 4$ on the same pair of axes.



The equation $x^2 = 4$ can also be written as $x^2 - 4 = 0$. It can be solved by graphing $y = x^2 - 4$ and $y = 0$ on the same axes.

18. What is another name for the line $y = 0$?

As shown in the figure, the graphs intersect in two points. This means that the quadratic equation $x^2 = 4$ has two solutions.



19. What are the two values of x that satisfy the equation $x^2 = 4$? Where do they appear in each of the two graphs above?

20. Explain why all of these quadratic equations are equivalent.

$x^2 = x + 6$

$x^2 - x = 6$

$x^2 - x - 6 = 0$

21. Graph the parabola $y = x^2$ and the line $y = x + 6$ on the same pair of axes. Label the points of intersection.

22. Graph the parabola $y = x^2 - x$ and the line $y = 6$ on the same pair of axes. Label the points of intersection.

23. Graph the parabola $y = x^2 - x - 6$ and $y = 0$ on the same pair of axes. Label the points of intersection.

24. Compare your answers to problems 21-23.

a. What is the solution to the quadratic equation $x^2 - x - 6 = 0$?

b. Which of the three graphs do you think gave the easiest way to find the solution to this equation?

25. Find the solutions to these equations by graphing a parabola and a line on the same pair of axes. As you saw in problem 24, there may be more than one possible pair of graphs that can be used. You may use any pair that will work.

a. $x^2 = 3x + 4$

b. $x^2 - 5 = -4x$

c. $2x^2 = 18$

DISCOVERY LAST DIGITS

26. What is the last digit for each of these numbers: 0^{100}, 1^{100}, 2^{100}, ..., 9^{100}? Most of these numbers have too many digits for the last one to appear in your calculator, so you will have to figure out some other approach. (Hint: Try finding the last digits of smaller powers of these numbers.)