LESSON 8.3

Slope

You will need:

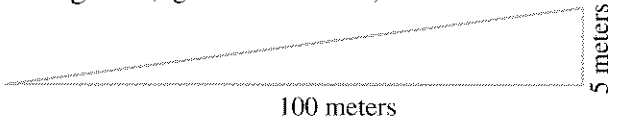
- geoboards
- dot paper

GRADE AND SLOPE

Steep roads sometimes have a sign indicating how steep they are. For example, the sign may say **5% Grade**. This means that you gain 5 units of altitude (the *rise*) for every 100 units you move in the horizontal direction (the *run*).

1. On a 5% grade, how many units of altitude do you gain for every
   a. 200 units you move in the horizontal direction?
   b. 25 units you move in the horizontal direction?
   c. 1 unit you move in the horizontal direction?

5% grade (figure is not to scale)



In math a 5% grade is called a *slope* of 0.05.

2. If the slope is 0.05, how many units do you move in the horizontal direction for every
   a. 30 units you gain in altitude?
   b. 0.05 units you gain in altitude?
   c. 0.5 units you gain in altitude?

3. The figure above is not to scale.
   a. What is the actual slope illustrated? (Use a ruler to measure the rise and the run.)
   b. Is it more or less steep than a 0.05 slope?

4. A sign in the mountains says **6% Grade. Trucks Use Low Gear.** Explain what a 6% grade is. Use the words *slope, rise,* and *run* in your answer.

5. In a nonmountainous area, the steepest grade allowed on a freeway is 4%. With this grade, how many meters of altitude do you gain per
   a. kilometer traveled in the horizontal direction? (A kilometer is 1000 meters.)
   b. meter traveled in the horizontal direction?

6. If you are climbing a mountain road with grade 5.5%, and you gain 1000 ft in altitude, how many miles have you traveled? (There are 5280 feet in a mile.)

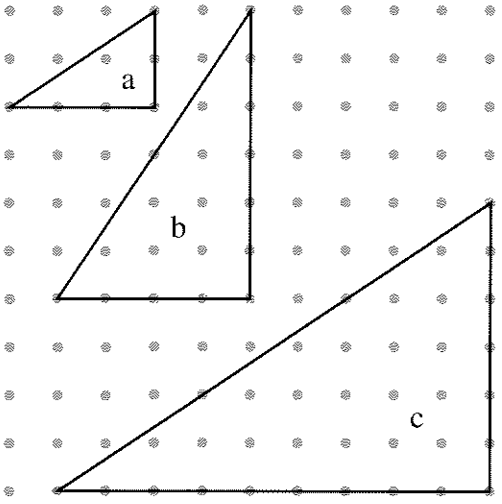
Definition: Slope is defined as the ratio of rise to run. $\text{slope} = \frac{\text{rise}}{\text{run}}$

7. a. How many units of altitude do you gain for every 100 units traveled on a horizontal road?
   b. What is the grade of a horizontal road?
   c. What is the slope of a horizontal line?

A horizontal road has grade 0. This is because no matter how much you move in the horizontal direction, you do not gain any altitude. The rise is 0 for any run. For example, for a run of 1, the slope is 0/1 which equals 0.

GEOBOARD SLOPE

The figure shows three geoboard right triangles. The side opposite the right angle in a right triangle is called the *hypotenuse.*



8. Find the slope of each hypotenuse in this figure.

9. How would you use the slope to find which hypotenuse is steeper? Which hypotenuses have the same steepness?

10. Two of the hypotenuses in the figure have the same slope. Explain why someone might make a mistake and believe all three have the same slope.

Do not use horizontal or vertical lines in problems 11-16. Start your lines at the origin.

11. a. What is the smallest slope you can find on a geoboard? Express it as a decimal.
b. Sketch a right triangle, like the ones above, to illustrate it.

12. Repeat problem 11 for the greatest possible geoboard slope.

13. Find a line having slope 1, and sketch several right triangles for it.

14. Find every possible geoboard slope that is a whole number.

15. Find every possible geoboard slope that is greater than 1 and less than 2. Express your answers as decimals.

16. Find every possible geoboard slope that is greater than 0.5 and less than 1. Express your answers as decimals.

SLOPES FROM COORDINATES

You may make a right triangle on your geoboard to help you answer the following questions.

17. What is the slope of the line joining
a. (0, 0) and (4, 5)?
b. (1, 1) and (5, 6)?
c. (0, 1) and (4, 5)?
d. (1, 0) and (5, 6)?

18. What is the slope of the line joining
a. (0, 0) and (8, 10)?
b. (0, 2) and (8, 10)?
c. (2, 3) and (3, 5)?
d. (4, 6) and (6, 10)?

For problem 19, you cannot use the geoboard.

19. What is the slope of the line joining
a. (23, 34) and (65, 54)?
b. (1.2, 3.4) and (5.6, 7.89)?

20. Generalization Explain how to find the slope of the line joining (a, b) and (c, d).

ROLLER COASTERS

Abe and Bea disagree about which roller coaster is steeper, the Plunge of Peril or the Drop of Death.

"The Plunge of Peril," according to the ad for the Great American Super-Park, "drops you 111 feet in seconds, with a mere 20 feet of horizontal displacement."

Abe and Bea have a photograph of themselves standing in front of the Drop of Death. They measured the roller coaster on the photograph, and got a drop of 10.1 cm for a run of 1.8 cm.

21. Use what you know about slope to help them decide which roller coaster is steeper. Explain your method.

22. **Project** The Plunge of Peril and the Drop of Death were invented for this lesson. Find the slopes of some real roller coasters.

DISCOVERY ***SLUMBER THEORY***

Number theory is the branch of mathematics that studies whole numbers and their properties. It has been the source of many challenging problems over the centuries. Slumber theory is a silly offshoot of number theory.

The key concept of slumber theory is that any whole number can be *sliced* into a sequence of whole numbers.

Example: 365 can be sliced in four different ways:

3 | 6 | 5; 36 | 5; 3 | 65; or 365.

(Note that the slices are indicated by a vertical slash. Note also that in slumber theory, not slicing is considered a form of slicing.)

23. How many ways are there to slice a four-digit number?

A number is *slime* if it can be sliced into a sequence of primes.

Examples: 5 is slime, since it is already prime. 2027 is slime (2 | 02 | 7)
4,155,243,311 is slime
(41 | 5 | 5 | 2 | 43 | 3 | 11)

24. Which one of the following numbers is slime?

a. 12 b. 345 c. 6789

25. 2 is the only even prime. Find the first three even slimes.

26. There are no prime squares. Find the first two slime squares.

27. There are no prime cubes. Find the first two slime cubes.

28. 2 and 3 are the only consecutive numbers that are both prime. Find the first three pairs of consecutive numbers that are both slime.

29. There is no triple of consecutive numbers that are all prime. Find the first two triples of consecutive numbers that are all slime.

30. Find the smallest number that is slime in more than one way. (In other words, it can be sliced into two different sequences of primes.)

31. Find the smallest number that is slime in more than two ways.

A number is a super-slime if you get a sequence of primes no matter how you slice it.

Example: 53 is a super-slime since 53 and 5 | 3 are both sequences of primes.

32. Find all the super-slimes.