# 10.3 Two Variables, Two Equations
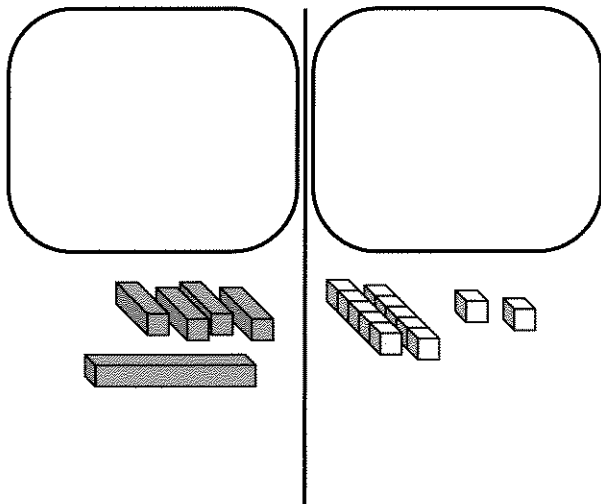
**You will need:**

the Lab Gear

## MYSTERY CONTAINERS

The Lab Gear may help you solve this problem.

1. A crate contains two small containers and three large containers. The total weight of the crate is 16 pounds.

   a. What are some possible weights of the small and the large containers? How many possible weights are there?

   b. Find the weight of four small containers and six large containers.

   c. Two containers are removed from the crate, and it is weighed again. Now it weighs ten pounds. Using this additional information, find possible weights for the small container and the large container. Comment on your answers.
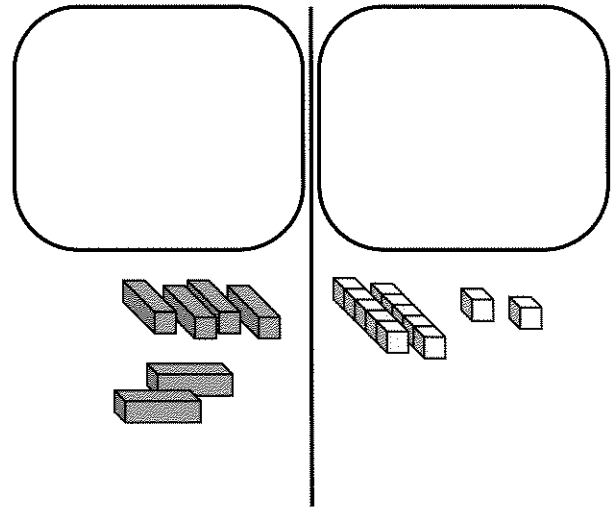
## ONE EQUATION, ONE CONSTRAINT

The workmat shows the equation

$$y + 4x = 12.$$



2. Using trial and error, find some values of $x$ and $y$ that make the equation true. (How many possible values are there?)

One of the $(x, y)$ pairs satisfying this equation also satisfies the constraint, or condition, that $y$ is twice $x$. If $y$ is twice $x$, then each $y$-block can be replaced with two $x$-blocks.



The resulting equation is $6x = 12$.

3. Solve for $x$ in the equation above. Then find the $(x, y)$ pair that satisfies both the equation $y + 4x = 12$ and the constraint that $y = 2x$.

For each problem, 4-7, model the equation on the workmat with the blocks. Then use the blocks to find an $(x, y)$ pair that satisfies both the equation and the constraint. Check your final answers in the original equations.

4. $4x - 7 = y + 3$
   Constraint: $y$ is two more than $x$.

5. $2y + x = 5$
   Constraint: $x$ is six less than $y$.

6. $2x + y = 9$
   Constraint: $x$ is three more than $y$.

**7.** $2y + x = 4$

Constraint: $x$ and $y$ add up to six.

For each problem you just solved, the constraint could have been written as an equation. For example, the constraint that the sum of $x$ and $y$ is six can be written $x + y = 6$. This means that in each of problems 4-7, you found an $(x, y)$ pair that *satisfied both of two given equations*. We say that you *solved* a system of simultaneous equations.

### SIMULTANEOUS EQUATIONS

Solve each system of simultaneous equations. If you want to use the Lab Gear, begin by modeling the first equation with the blocks. Then use the second equation to substitute blocks for the $y$-blocks or for the $x$-blocks. Check your answers.

**8.** $\begin{cases} 2x - y = 2 \\ y = 3x \end{cases}$     **9.** $\begin{cases} 4x + y = 10 \\ y = 6x - 20 \end{cases}$

**10.** $\begin{cases} x - 4y = 23 \\ x = -5y - 4 \end{cases}$     **11.** $\begin{cases} 3y + 2x = 7 \\ 3y = 4x - 5 \end{cases}$

### MIND READING

What numbers am I thinking of?

**12.** Their sum is 7. Their difference is 3.

**13.** Their sum is 18. The second is twice as large as the first.

**14.** The first minus the second is 3. Twice the first, minus twice the second is 6.

**15.** ☞ One of problems 12-14 has more than one answer. How many answers does it have? Why?

### REVIEW/PREVIEW  EVALUATING

**16.** Two $(x, y)$ pairs that satisfy the equation $2x + 3y = 16$ are given in the table below. Copy and complete the table.

| $x$ | $y$ | $2x + 3y$ | $x + y$ | $x - y$ | $4x + 6y$ | $x + 1.5y$ |
|---|---|---|---|---|---|---|
| -1 | 6 | 16 | 5 | — | — | 8 |
| 2 | 4 | 16 | — | -2 | 32 | — |
| – | 5 | 16 | — | — | — | — |
| – | -6 | 16 | — | — | — | — |
| – | – | 16 | — | — | — | — |
| -4 | – | 16 | — | — | — | — |

**17.** ☞ Study the table you made. In which columns are all the values the same? Why?

### REVIEW/PREVIEW  SOLVING FOR y

Set up these problems with the Lab Gear, and rearrange the blocks so that $y$ is by itself on one side of the equation. Write equations to show your steps. In some cases, you will need to finish the problem without the blocks.

**18.** $-4x + y = 6$     **19.** $4x + 2y = 10$

**20.** $-6x + y = 4$     **21.** $-6x + 3y = 9$

**22.** $6x - 3y = 12$     **23.** $x + 2y = 8$

**24.** $x - y = 1$     **25.** $6x - 5y = 0$

**26.** ☞ Explain how to solve for $y$ (without the Lab Gear), with the help of an example.

**27.** Generalization Solve for $y$.

$$Ax + By = C$$

**DISCOVERY** *CONSECUTIVE NUMBERS*

**28.** Compute, and look for a pattern.
  a. $1 \cdot 2 \cdot 3 + 2$
  b. $2 \cdot 3 \cdot 4 + 3$
  c. $4 \cdot 5 \cdot 6 + 5$
  d. $(5 - 1) \cdot 5 \cdot (5 + 1) + 5$
  e. $9 \cdot 10 \cdot 11 + 10$
  f. $(10 - 1) \cdot 10 \cdot (10 + 1) + 10$

**29.** ♀ Use algebra to explain the pattern.

**30.** The product of three consecutive numbers divided by their sum is 1. What are the numbers?

**31.** Repeat problem 30, if the product divided by the sum is the following:
  a. 5          b. 16

**32.** ♀ What can you say about the middle number if the product of three consecutive numbers divided by their sum is a whole number?

**DISCOVERY** *NINE FACTORS*

**33.** **Project** The number 1 has one whole number factor, itself; 2 has two factors, 1 and 2; 3 has two factors; and 4 has three factors. (What are they?) Find some numbers having *nine factors*. Explain.