LESSON 11.6

# What is Probability?

**You will need:**

two coins

This lesson will introduce you to three interpretations of probability.

## RELATIVE FREQUENCY

While waiting for his food at the Slow Food Café, Zoltan asked himself, "What is the probability of getting at least one head when tossing two coins?" He thought it might be 1/2, since there was an equal chance of getting heads or tails, or 1/3, since there were three possibilities (two heads, one head, no heads). He decided to find out by doing an experiment. Here are his notes on the first eight tosses (or *trials*).

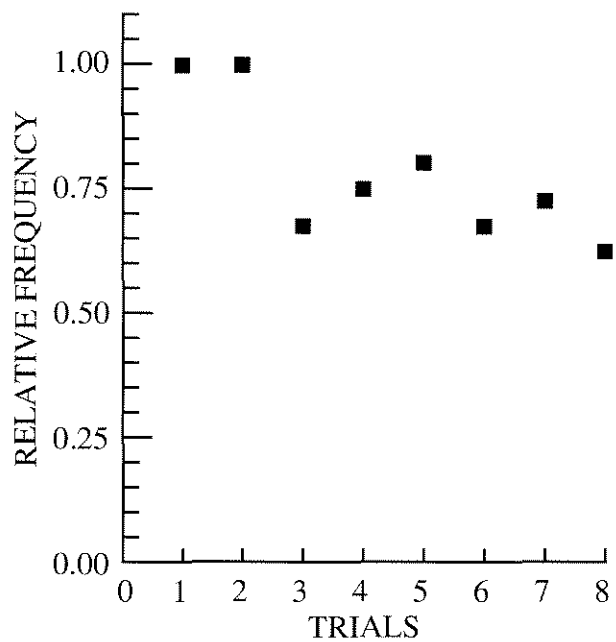
HH, HT, TT, HT, HH, TT, TH, TT...

He made a table of the results. A *success* is a toss where one or two heads appeared.

**Definition:** The *relative frequency* of the successes is the ratio of successes to trials.

| Trials so far | Successes so far | Relative frequency |
|---|---|---|
| 1 | 1 | 1/1 = 1.00 |
| 2 | 2 | 2/2 = 1.00 |
| 3 | 2 | 2/3 = 0.67 |
| 4 | 3 | 3/4 = 0.75 |
| 5 | 4 | 4/5 = 0.80 |
| 6 | 4 | 4/6 = 0.67 |
| 7 | 5 | 5/7 = 0.72 |
| 8 | 5 | 5/8 = 0.63 |

He graphed the results, with relative frequency on the $y$-axis, and trials on the $x$-axis.



1. Toss a pair of coins 30 times. Make a table like Zoltan's.

2. Make a graph like Zoltan's for the data in your table.

3. If you tossed the coins 100 times, what do you think your graph would look like? What if you tossed them 500 times? Explain.

**First Definition:** The *probability* of an event is often interpreted to mean the relative frequency with which that event occurs if the experiment is repeated many, many times.

**Example:** If you roll a die many times, you expect the relative frequency of threes to be approximately 1/6.

4. Explain why the relative frequency of an event is a number from 0 to 1.

## EQUALLY LIKELY OUTCOMES

This definition is the most common interpretation of probability.

**Second Definition:** The *probability* of an event $A$ is

$$P(A) = \frac{e}{t}$$

where:

$e$ = the number of equally likely outcomes in the event.

$t$ = the total number of equally likely outcomes possible.

**Example:** In the two-dice experiment, say that event $D$ is the event that the sum is 8. Then

$$D = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

Since $D$ consists of five equally likely outcomes, and the total number of equally likely outcomes is 36,

$$P(D) = \frac{5}{36}.$$

5. For the two-dice experiment, find an event having the following probabilities:

   a. $\frac{2}{36}$ b. $\frac{1}{12}$

6. For the two-dice experiment, find the probability of these events.

   a. The product is more than 25.

   b. The product is less than 50.

   c. The sum is 7 or 11.

7. Explain why any probability $p$ will always satisfy the inequality $0 \leq p \leq 1$.

8. For the two-dice experiment, find an event having the following probabilities:

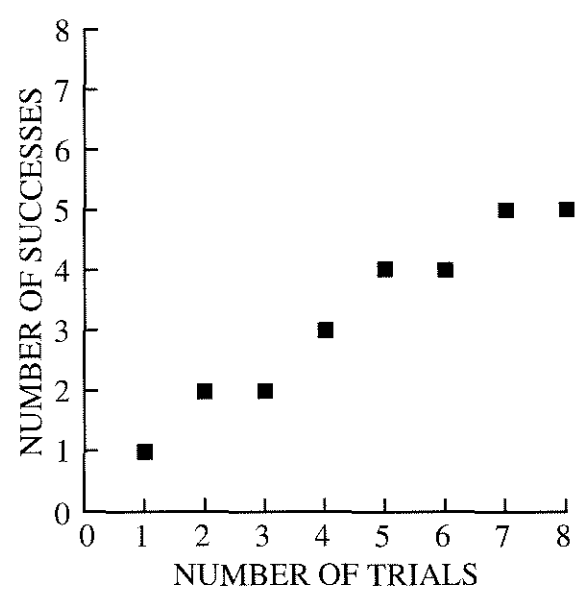
   a. 0 b. 1

9. List all the equally likely outcomes in Zoltan's two-coin experiment. (Hint: Think of the coins as a penny and a nickel. Make a table.)

10. What is the probability that there will be at least one head when tossing two coins? Explain.

## THEORETICAL vs. OBSERVED PROBABILITY

Zoltan graphed his results another way. This time he put the number of successes on the $y$-axis and the number of trials on the $x$-axis.



**11.** Make a graph like Zoltan's for the data in the table you made in problem 1.

**12.** On your graph, draw lines having equations:

$$\textit{successes} = \textit{trials}$$
$$\textit{successes} = 0.75 \cdot \textit{trials}$$
$$\textit{successes} = 0.67 \cdot \textit{trials}$$
$$\textit{successes} = 0.50 \cdot \textit{trials}$$

**13.** What do rise and run each measure on this graph? What does slope represent?

On a graph like this, the *theoretical probability*, as predicted by the analysis of equally likely outcomes, can be represented as a line through the origin, having slope equal to the probability. The *observed probability* as seen in the experiment is represented by the slope of the line through the origin and the corresponding data point. Note that data points rarely land exactly on the theoretical line.

**14.** Which line that you drew in problem 12 represents the theoretical probability? Explain.

**15.** Add a line representing the theoretical probability to the graph you made in problem 2. Explain.

## SUBJECTIVE PROBABILITY

A third interpretation of probability is *subjective probability*. This is the probability that a person assigns to an event based on his or her own knowledge, beliefs, or information about the event. Different people may assign different probabilities to the same events.

**Example:** Before Mark took his driving test, Karen said, "I think you've got about a 60% chance of passing."

What subjective probability would you assign for each of the following events? Explain your reasons.

**16.** It will be cloudy on a night with a full moon this month.

**17.** You will be assigned no math homework this Friday.

**18.** School will be cancelled next week due to bad weather.

**19.** Exactly half of the students in your math class next year will be boys.