

Representing Functions

You will need:

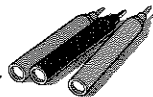
graph paper



function diagram paper



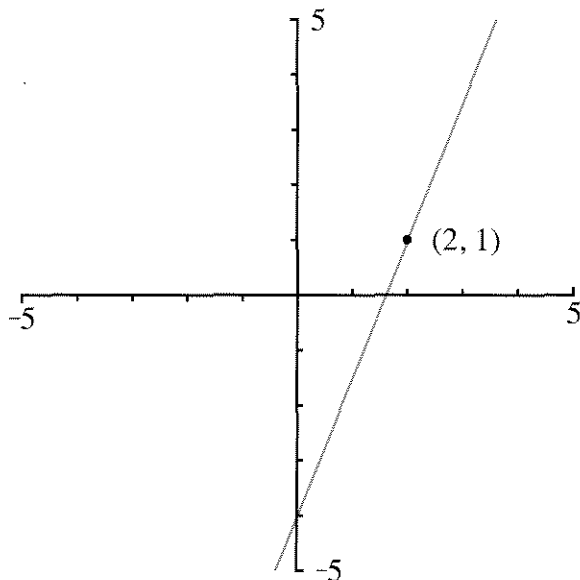
colored pens



POINTS AND LINES

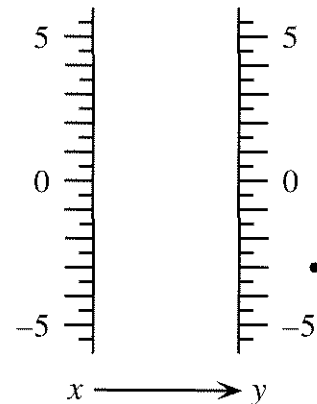
As you know, an (x, y) pair is represented as a point on a Cartesian graph and as an in-out line on a function diagram. In this section we will review how an equation of the form $y = mx + b$ is represented in these formats.

1. For the function represented by this Cartesian graph,
 - a. write the equation;
 - b. draw a function diagram.



2. Extend the in-out lines in the function diagram you made in problem 1. They should meet in one point, called the focus.
3. What is the minimum number of lines you need to draw to find the focus? Explain.

Actually, a function of the form $y = mx + b$ can be represented by just the focus, as you will see in the next problem.



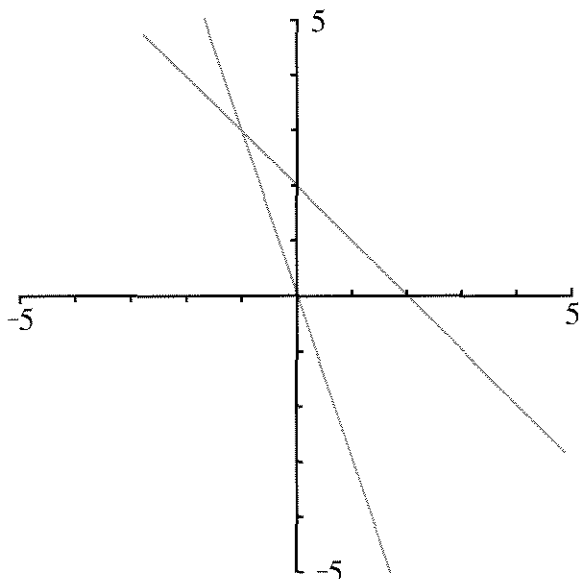
4. The figure shows the focus of a certain function of the form $y = mx + b$.
 - a. Place a ruler on the focus, and find three in-out lines. Do not draw the lines, but keep a record of the (x, y) pairs.
 - b. Find the equation.
5. If you were to make a Cartesian graph of this function, what is the minimum number of points you would need to plot? Explain.

This table shows how points and lines appear in the two representations. Notice how points and lines are switched when going from one representation to the other.

Object	Representation	
	on Cartesian graph	on function diagram
(x, y) pair	one point	one line (in-out)
linear equation	one line	one point (the focus)

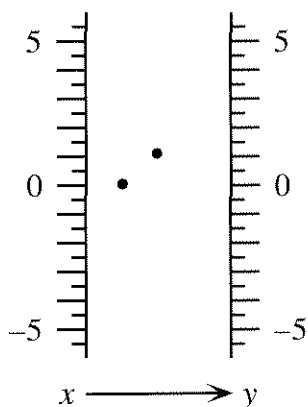
FOCUS ON SIMULTANEOUS EQUATIONS

This graph shows $y = -3x$ and $y = -x + 2$.



6. What point do the two lines have in common?

The function diagram shows the foci of $y = -3x$ and $y = -x + 2$. (Foci is the plural of focus.)



7. Check that the foci are placed correctly.
- Place a ruler on the focus and find three in-out lines for each function. Do not draw the lines, but keep a record of the (x, y) pairs.
 - Check that the (x, y) pairs you found satisfy the equations.

8. If you were to draw an in-out line containing both foci, what (x, y) pair would it represent?
9. How is the solution of a system of linear equations represented on:
- a Cartesian graph?
 - a function diagram?
10. Explain how to solve this system by using a function diagram. (Hint: First find each focus, then find the solutions.)

$$\begin{cases} y = 0.5x + 2 \\ y = 2x - 1 \end{cases}$$


Because two lines meet in a point, the solution to a system of simultaneous equations is represented on a Cartesian graph by a point. Because two points determine a line, the solution to a system of linear equations is represented on a function diagram by a line (an in-out line).

FAMILIES OF FUNCTIONS





Definition: A *family* of functions is a group of functions that share a certain attribute.

11. All functions having equations of the form $y = 5x + b$ belong to the $m = 5$ family.
- Sketch the graphs of two members of the family.
 - What do all graphs for this family have in common?
12. All functions having equations of the form $y = mx + 7$ belong to the $b = 7$ family.
- Sketch the graphs of two members of the family.
 - What do all graphs for this family have in common?

All functions in the same b -family have foci that lie on the same in-out line.

13. These four functions are in the same b -family. For each one, draw in-out lines to find the focus and mark it with a colored pen or pencil. Do all four on the same diagram.
- a. $y = 0.5x - 2$ b. $y = 2x - 2$
 c. $y = -2x - 2$ d. $y = -0.5x - 2$
14. What is the family name for the functions in problem 13?
15.  Why do all the foci of the functions in problem 13 lie on the same in-out line? Which in-out line is it? Explain.
16. The foci for all functions in the family $b = -3$ also lie on one in-out line. Which line? Explain how you know.

Many m -families also have foci that lie on the same in-out line in a function diagram.

17. a. On a function diagram, find and mark the focus for $y = -2x + 3$.
 b. On the same function diagram, find and mark the focus for $y = -2x + 1$.
 c. Find and mark the focus for several other graphs of the form $y = -2x + k$.
18.  What is the family name for all the functions in problem 17? Explain why the foci are all on the same line. Describe the line.
19.  If two functions both have a focus on the same vertical line, what would their Cartesian graphs have in common?
20.  What is the family name for all functions having focus half-way between the two number lines?
21. There is one m family for which the function diagrams have no focus, because the in-out lines do not meet. Which m family is this?
22. **Summary** On a function diagram, what is true of the foci of all linear functions in the same
 a. m family? b. b family?
23.  The functions representing Charles's Law for gases in the graph in Lesson 4 form a family that is neither an m nor a b family. If you were to make function diagrams for them, the foci would all be on a certain in-out line. Which one? Explain.