LESSON 13.1

Rectangular Pens: Constant Perimeter

You will need:

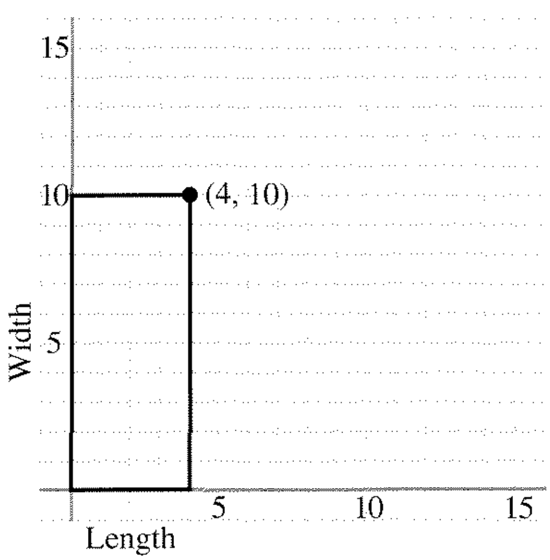
graph paper

a graphing calculator (optional)

1. **Exploration** You want to make a rectangular pen for Stripe, your pet zebra. Even though Stripe takes many walks around town, you want to make sure she has as much space as possible inside the pen. You have 50 feet of fencing available. If you use all of it to make the pen, what is the biggest area possible? Find out by trying various dimensions for the pen.

WIDTH AS A FUNCTION OF LENGTH

You have 28 feet of fencing to make a rectangular pen. There are many possible dimensions for this pen. One possible pen, 10 feet wide by 4 feet long, is shown below. In this section you will investigate how the length and width change in relation to one another if you keep the perimeter constant.



2. a. On graph paper, draw axes and at least six pens having a perimeter of 28.

b. The upper right corner of the pen in the figure has been marked with a • and labeled with its coordinates. Do this for the pens you drew. Then connect all the points marked with a •. Describe the resulting graph.

3. a. Make a table showing all the coordinates on your graph. Look for a pattern and make three more entries in the table.

b. Write an equation for the function described by your graph and table.

Summary

4. The point whose coordinates are (4, 10) is on the graph.

a. What does the sum of these numbers represent in this problem?

b. What does the product represent?

5. a. What is the greatest possible length of a pen? How can you see this on your graph?

b. How many rectangles are possible if the dimensions are whole numbers? How many are possible otherwise?

c. Explain why the graph should not be extended into quadrants II and IV.

6. If you increase the length by one foot, does the width increase or decrease? Does it change by the same amount each time? Explain.

AREA AS A FUNCTION OF LENGTH

In the previous section you may have noticed that the area of the rectangles changed even though the perimeter remained constant. In this section you will investigate how the area changes as a function of length, if you keep the perimeter constant.

7. Write the area of the corresponding rectangle next to each of the points marked with a • on the graph from problem 2.

8. Make a graph of area as a function of length. Show length on the x-axis and area on the y-axis. Connect the points on your graph with a smooth curve. What kind of curve is it?

9.

a. Label the highest point on your graph with its coordinates. Interpret these two numbers in terms of this problem.

b. Where does the graph cross the x-axis? What do these numbers mean?

c. If you increase the length by one foot, does the area increase or decrease? Does it change by the same amount each time? Explain.

10. Summary

a. Describe in words how you would find the area of the rectangular pen having perimeter 28 if you knew its length.

b. If the perimeter of a rectangular pen is 28 and its length is L, write an algebraic expression for its area in terms of L.

c. If you had 28 feet of fencing and wanted to make the largest possible rectangular pen, what would its length, width, and area be? Explain.

Generalizations

11. Say the perimeter of a rectangle is P and its length is L. Write the following expressions in terms of P and L. (A sketch may help.)

a. an expression for the width

b. an expression for the area

12. Explain how to find the length that gives the maximum area. Write an algebraic expression for it in terms of P only.

PARABOLAS THROUGH THE ORIGIN

13. Graph each of the following functions, using graph paper. Since you will want to compare your graphs in the end, use the same pair of axes for all your graphs. Use a scale that will show values from -5 to 20 for x and from -20 to 100 for y. This will allow you to see all four graphs clearly.

a. $y = x(8 - x)$ b. $y = x(15 - x)$

c. $y = x(12 - x)$ d. $y = x(20 - x)$

14. For each of the four parabolas in problem 13,

a. label the graph with its equation;

b. label the x-intercepts;

c. label the vertex.

15. Generalization

a. Describe the graph of a parabola having equation $y = x(b - x)$. Write expressions for the coordinates of its intercepts and vertex in terms of b.

b. Do these expressions work for negative values of b? Explain, using examples.

16. Graph.

a. $y = x(x - 8)$ b. $y = x(x - 15)$

c. $y = x(x - 12)$ d. $y = x(x - 20)$

17. How do the graphs differ from the ones in problem 3? Discuss the vertex and the intercepts.

18. Generalization

a. Describe the graph of a parabola having equation $y = x(x - q)$. Write expressions for the coordinates of its intercepts and vertex in terms of q.

b. Do these expressions work for negative values of q? Explain, using examples.

19. Graph $y = ax(x - 3)$ for:

a. $a = 1$ b. $a = -1$

c. $a = 2$ d. $a = -3$

20. What is the effect of a on the position of:

a. the vertex?

b. the x-intercepts?

Find equations of the form $y = ax(x - q)$ for parabolas *through the origin*, with the given x-intercept and the vertex with the given y-coordinate.

		x-intercept	**y-coordinate of vertex**
21.	a.	4	4
	b.	4	8
	c.	4	2
	d.	4	-6
22.	a.	8	4
	b.	2	4
	c.	-4	4
	d.	-6	-6

REVIEW *FIXED POINTS*

23. Find the fixed point for the function $y = 6x + 8$.

24. Solve the system: $\begin{cases} y = 6x + 8 \\ y = x \end{cases}$

25. Explain the statement: *To find the fixed points of a function, find the intersection of its graph with the line $y = x$.*

26. Test whether the statement is true by finding the fixed points of $y = x^2$.