PATHWAYS

A GUIDE TO CREATING YOUR OWN ALGEBRA COURSE

Dave Logothetti, a charismatic teacher who wrote and spoke about the teaching of problem solving, gave a talk about curriculum for algebra at a California Math Council conference several years ago. "Make a list of topics in your text that you find interesting," he said. "Make another list of topics you think your students will find interesting. There should be some overlap; just teach that." The nervous laughter that resulted reflected the difficulty that we teachers have in letting go of the traditional curriculum we grew up with. But Logothetti understood that having students study a few topics that really engaged them in thinking mathematically had far more value than giving them superficial command of a large body of techniques. There was little danger then that teachers would find too many topics in their texts that met Logothetti's two criteria!

With *Algebra: Themes, Tools, Concepts* we now have a text that offers far more to engage students and teachers than can be studied in a one-year course. Exploring, investigating, explaining, and generalizing take time, so difficult choices must be made about which topics to study. The purpose of these "Pathways" is to make it easier for you to construct a one-year course using the rich resources of this text to meet the needs of each group of students, while reflecting your own priorities and interests.

USING THE PATHWAYS

If you start from the beginning of the text doing those sections that seem most worthwhile and skipping those you find less appealing, it is likely that material in the later chapters which you may consider crucial will get skipped by default. The Pathways provide carefully chosen assignments that lead through the core material in an efficient way so that you can make effective choices between adding activities that enrich core material, and moving on to other topics. The Pathways consist of a basic path, a basic-plus path, additional strands, and alternate paths. For the basic and basic-plus paths, suggested classwork and homework assignments are provided for each day.

The basic path is a suggested "skeleton" from which to build a year-long course. It includes essential topics as well as those that reflect the unique strengths of the text. Topics about which there is debate concerning their inclusion in first-year algebra have been left out, as have the geometry lessons, except those needed to develop understanding of included algebra topics. Classwork and homework problems have been chosen to focus on the central concepts and to allow completion of the lessons in about 76 class days, so that plenty of time is left for additional topics.

The basic-plus path provides one option for expanding the basic path into a "core" course which may appeal to many teachers. It strikes a middle path on inclusion of factoring, embraces the text's emphasis on graphing (but does not assume the availability of graphing calculators), and takes advantage of the unique strengths of the Lab Gear approach. Its 130-day estimate leaves time for you to include additional enrichment topics.

The alternate paths suggest two ways to alter the basic-plus path to create core courses with particular emphases in either graphing or applications. Each of the alternate paths can be completed in approximately 130 class days.

The additional strands are sequences of text-book sections, each of which adds or goes deeper into a specific topic which you may wish to include or emphasize. The topics covered are: quadratic equations and functions, factoring, exponential growth, sequences, square roots, geometry, probability, and abstract algebra. They can be used to make adjustments to the basic-plus, graphing, or applications paths, or added at the end if time permits.

ESTIMATING TIME

Predicting how many days will be required to complete a given lesson or sequence of lessons is at best a fine art, and at worst a very crude science, but it is nonetheless essential to effective course planning. Variables include length of the class period, expected daily homework preparation time, amount of time allowed for homework discussion, frequency of quizzes and tests, and amount of time allowed for them, the ability and background of the students, and the size of the class. You will quickly see how the time estimates given in the basic-plus path compare to your actual teaching time. The estimates for the basic, basic-plus, and alternate paths include approximately one "wild-card" day for each three days of planned class-work. A wild-card day allows time for quizzes, tests, catch-up, review, group presentations, or optional enrichment. Even so, the 130-day estimate is plenty to begin with for one-year courses meeting every day. The basic-plus path and alternate paths omit many appealing lessons which reinforce and extend the concepts studied, and you'll want to do some of these if time permits.

MAKING ADJUSTMENTS

Wherever possible, the suggested classwork in the basic-plus path concludes with problems which can be finished for homework, as this enhances flexibility and encourages classroom productivity. But you will want to be careful not to assign as homework those problems that many students won't really grasp without discussion. It pays to stockpile homework problems from previous lessons to be used when not enough progress is made through the current lesson to assign the homework on it. These may be problems from the "review" or "discovery" sections which come at the end of many lessons, from the Essential Ideas section of the current or preceding chapter, or problems that you may have omitted from a previous assignment due to time considerations. For those lessons that clearly need two days in class before homework from the text can be assigned, there are problems in the Extra Practice section of this binder which can be used as homework to reinforce the first day's work.

Suggested homework for the first few sections of a chapter often is taken from the Essential Ideas section in the preceding chapter. This assumes that you are beginning the new chapter before giving a test on the preceding one. If you prefer to conclude a chapter with a test before moving on, the Essential Ideas can form the basis for a day or two of review, combined perhaps with discussion of reports the students have written or other enrichment work. In this case, the Essential Ideas homework assignments can be replaced with test corrections, or with "take-home" test problems that require more time than in-class testing allows. The "additional problems" in the Test Bank section of this binder are often good for such assignments.

The Basic and Basic-Plus Paths

The basic path includes the following essential topics of a first-year algebra course as well as those topics that reflect the unique strengths of the text:

- Investigation, data collection, and graphing.

- Simplifying variable expressions with Lab Gear and with symbols, including minus and opposites, distributive property, multiplication, and division.

- Powers: exponents and scientific notation, introduction to exponential growth, properties of exponents.

- Function diagrams and working backwards.

- Functions and graphs: rate, time, and distance; direct variation; step functions and piecewise equations; interpreting graphs; and solving with graphs.

- Division, reciprocals and rational expressions.

- Using rational expressions: proportions and averages, percent increase and decrease, midpoints and averages, unit conversions.

- Comparison: differences and ratios

- Introduction to inequalities.

- Linear equation solving with Lab Gear and with symbols: equivalent equations, transformations, identities, and "no solution."

- Linear modeling and applied equation solving.

- Linear functions: function diagrams and Cartesian graphs, magnification and slope, equations of lines.

- Distance and absolute value.

- Square roots, distance, and the Pythagorean Theorem.

The basic-plus path includes all the topics of the basic path and adds the following:

- Use of the Lab Gear to study factoring of simple quadratic expressions, as a way to provide insight into the structure of algebra.

- Use of both Lab Gear and graphic approaches to solve simple quadratic equations and to explore completing the square.

- More work with area on the geoboard.

- Review exercises on signed numbers and fractions.

- Linear systems, solved graphically and by substitution.

- Negative exponents and simple exponential functions.

- Introduction to probability.

- Similarity.

- Expanded study of linear functions, rational expressions, square roots, and using and interpreting graphs.

The chart on the following pages provides a day-by-day plan for classwork and homework assignments for all the lessons in the basic-plus plan. The lessons that are not included in the basic path are marked with a "+" sign. Many of the homework assignments require paper HomeWork Gear, which is available from Creative Publications or can be made by copying the page of Lab Gear blocks in the Support Masters section of this binder. Those homework assignments marked "P" are found in the Extra Practice section of this binder. For each lesson marked "M" there is a page in the Support Masters section of the binder that may assist you and your students in completing that lesson. The symbol "G" indicates problems that can be done effectively with a graphing calculator. Some of these graphing problems are already in the basic-plus assignments, and others are recommended for inclusion if graphing calculators are available.

The Basic and Basic-Plus Paths

Time estimates:
- Basic path = 57 days + 19 "wild card" days = 76 total class days.
- Basic-plus path = 98 days + 32 "wild card" days = 130 total class days.

Day	Lesson	Topic	Classwork	Notes	Homework
1	1.1	polyominoes: investigation, data collection	1–3, 5, 6	Begin 1.2: 1 in class.	1.1: 7–9 1.2: 1M
2	1.2	introduction to graphing	10–14, 6–9M	15–20: Discuss only.	1.2: 21–22
3	1.3	Lab Gear introduction	1–9	Discuss only.	1.3: 11 1.A: 1–6M
	1.4	substituting and evaluating	6–11	Discuss 1–5 first.	
4	1.5✦	dimension	6, 7, 11, 13, 14–18	Discuss 1–5 first.	1.A: 7
5	1.7✦	perimeter	4–9, 11, 12		1.7: 21–27
6	1.9	Lab Gear multiplication	2–11, 18		1.5: 19–21 1.6: 14–17, 21
7	1.12✦	geoboard area	4–17		p.39: 4–8, 12, 17
1	2.1	minus and opposite with Lab Gear	1–6		P 2.1, 2.2: 1–10
2	2.2	adding zero	1, 7–12		P 2.1, 2.2: 11–13 2.2: 2–5, 14, 15
3	2.3	multiplication	2–4, 6–8, 11–15		pp.40–41: 19–21
4	2.4	distributive law	1–6, 9, 12bc, 13, 15, 16		2.A
5	2.5	powers	1–5M		2.5: 8–13M 2.6: 28
6	2.7	function diagrams	1–3, 9–12		2.7: 7, 14–16, 4–6
	2.8	rate, time, distance diagrams	1–3M		
7	2.9✦	function diagrams: inverse operations	1–8, 10–15M	P 2.9: A good summary.	p.78: 1–6
8	2.12✦	geoboard triangles	1–9M		2.12: 10–13
1	3.1	working backwards	2–11		3.1: 12
2	3.2✦	negative numbers, Cartesian graphs and vocabulary	1, 3–23		Finish classwork.

✦ Lesson not in basic path   P Extra Practice page   M Support Master page   G Graphing calculator problems

Day	Lesson	Topic	Classwork	Notes	Homework
3	3.3	distributing minus	1–5, 7–9		3.3: 10–12, 14–15
					P 3.3: 1–4
4	3.3	distributing minus	16–20		3.3: 21–25,
					P 3.3: 5–8
	3.5	introduction to inequalities	3.5: 1, 2, 4, 5		P 3.5: 5–10
			P 3.5: 1–4		3.5: 18–21
			3.5: 14–17		
5	3.5	introduction to inequalities	22–24		3.1: 13–14
6	3.6	division, and multiplication with tables	1–9acd, 10, 13–19		3.7: 1–4
					P 3.6: 1–5
7	3.7	reciprocals	5–7, 10		P 3.7: 1–8
8	3.7	reciprocals	18–27		3.8: 1
9	3.8✦	C° versus F°: introduction to linearity, data	1–8M		3.8: 11–17
10	3.9	equation solving: cover-up method	1–8	P 3.9: More cover-up practice.	Finish classwork.
11 12	3.12✦	equivalent fractions and similarity	1–7, 10–16		3.C: 1–11 (2 nights)
1	4.1	rate, time, distance, and graph reading	1, 3–6, 8–13M		Finish classwork.
2	4.2	representations of functions	1–6, 8–10		Finish classwork.
3	4.2	representations of functions	11–13		pp.119–120: 8–18
	4.3	−x² vs (−x)²	2–6M	G: 1, 7–15	
4	4.4	intercepts	1–9	G: 10–18, 21–23	4.4: 19–20
					p.120: 26–31
5	4.A✦	graph reading	1–4		4.A: 5–9
6	4.5	ratio, proportion, lines	1–7	G: 1,2	4.6: 1–6
7	4.5	ratio, proportion, lines	8–16, 19–20		Finish classwork.
8	4.6	direct variation	7–23		Finish classwork.
9	4.10	graph reading	1–10, 16–20M		4.6: 24
					4.10: 22,23
10	4.11	step functions, horizontal and vertical lines	1–10		p.163: 1–6
11	4.11✦	graph reading	12–15		p.160: 4, 6–12, 18

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Day	Lesson	Topic	Classwork	Notes	Homework
1	5.1✚	constant sums	1–5, 13–16		5.1: 17–20
2	5.2✚	constant products	1–9, 12, 13	G: 12	Finish classwork.
3	5.2✚	graphing errors	14	14: Discuss only.	5.3: 20–25 P 5.3: 1–6
	5.3	rational expressions	12–14, 16–18		
4	5.4✚	factoring	1–8		5.4: 15, 17, 20, 24 P 5.4: 1, 2
5	5.4✚	factoring	16, 18, 21abc	Review table multiplica-tion.	P 5.4: 3, 4
	5.6✚	factoring	2, 3, 7–9, 11		
6	5.6✚	factoring	13–16, 19–23	Review factor trees.	P 5.6: 1–5 5.6: 17, 18, 24, 25
1	6.1	using graphs, piece-wise equations	1, 3, 6–11		6.1: 16–18 p.201: 5, 9, 10
2	6.1	using graphs, piece-wise equations	13–14		6.2: 11–13 5.7: 24
	6.2	inequalities, simplifying with parentheses	4, 6, 7, 9, 10		
3	6.2	inequalities	18, 20, 21		6.2: 14–15, 19, 22–23
	6.3	solving with Lab Gear	4–7 P 6.3: 1–11 odd		p.201: 13–15 P 6.3: 2–12 even
4	6.3	transforming equations with Lab Gear	P 6.3: 13, 14–22 even		P 6.3: 15–23 odd 6.3: 12–19
5	6.4	identities, no solution	1, 10, 12abce, 16, 17		6.4: 23 6.A: 1–3
6	6.5	solving by graphing	1–9	G: 10–17	6.A: 4 6.5: 10ad, 11ad
7	6.6	equivalent equations, applied equation solving	12–18		6.5: 12 6.6: 19, 20, 22, 25–27
8	6.7	ratio and difference	1–4, 7, 8, 10, 12, 14–19, 21		Finish classwork.
9	6.8	equivalent equations	5–8, 10, 11, 14, 15, 17	Discuss 13.	6.8: 9, 12, 16, 18, 23, 24 P 6.8: 1–5
10	6.9	rational expressions	1, 5–8		6.9: 10–14 6.8: 25–27

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Day	Lesson	Topic	Classwork	Notes	Homework
11	6.10	using rational expressions	1–9		6.10: 13–18
12	6.12	preparation for square root,	6, 9, 10, distance	G: 23–28 11–13, 15	p.242: 5, 6, 8, 9
1	7.1+ 7.2+	squaring binomials applied squaring	1 1–3, 7M		7.2: 4–6, 8, 9M
2	7.3+	Lab Gear squares	2, 3, 8, 9, 12–14M		7.3: 16–18 7.4: 2–5
3	7.4+	difference of squares	6, 7, 8ac, 9–12		7.4: 21, 29 P 6.8: 6–10
4	7.4+ 7.5+	difference of squares identities with squares	14–16 11, 12		7.5: 13
5	7.6+	solving quadratics with graphs	1, 2, 7, 8, 11	G: 1–19	7.4: 23, 25, 31 7.5: 26
6	7.7+	solving quadratics with equal squares	1–4, 6–13	G: 1–5	7.7: 5 7.8: 1, 2
7	7.8	exponents (including 0)	4–11		7.8: 13–15, 19–23
8	7.9	scientific notation	2, 5–8, 12, 13		7.7: 29, 31 7.9: 10, 11
9	7.10	exponents and calculators	1–3, 6–11		7.11: 1, 2, 5, 6, 8, 9
10	7.12	square roots and distance	1–5, 7, 8, 15	Review 6.12: 10, 11	p.282: 1, 2, 21 8.1: 1–4
1	8.1	introduction to linear growth	8, 10–12	P 8.1: Motivates work on functions.	P 8.1: 1–3 8.2: 1–3
2	8.2	linear function diagrams	5		8.2: 6, 7 8.3: 1–3
3	8.2	linear function diagrams	8–10, 16–21	12–15: Nice for enrichment.	8.2: 22
4	8.3	slope	4–13, 15, 18–21		Finish classwork.
5	8.4	linear functions	2–14	G: 1	8.A: 1–4, 7 P 8.4 : 1
6	8.4	linear functions	15–27	Discuss last part of 28.	8.4: 28–30 8.A: 9
7	8.5	exponential growth, properties of exponents	1–9		8.5: 10–16

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Day	Lesson	Topic	Classwork	Notes	Homework
8	8.6+	exponential growth	1–5		8.7: 3
					P 8.4: 2, 3
	8.7	percent increase	1, 2, 4, 5, 7		
9	8.7	percent increase	8, 9, 11, 12, 15, 17		8.7: 10, 16, 20–26
10	8.8	percent decrease	1–8, 10, 11, 13		8.7: 27, 28, 30
					P 8.7, 8.8: 1–4
11	8.9	properties of exponents	1–6, 17–19		8.9: 7–12, 20
12	8.10+	properties of exponents	1, 3, 7–12ace, 14, 16–18		8.10: 12bdf, 19, 22, 25, 27
					8.B: 1,2
13	8.11+	negative exponents	1–9		8.11: 14, 15, 18–20, 23, 24, 29
14	8.12+	scientific notation	1–7, 9–12		8.12: 14,15
					8.C: 2
					p.323: 8, 10
1	9.1	distance, absolute value	1, 2, 5–7, 11–19		p.324: 12–15
2	9.2	Pythagorean Theorem	4–9	Begin with definitions.	9.2: 10–12
					p.324: 18, 22, 26, 27
3	9.6	midpoints and averages	1, 4, 5, 7–12, 17		9.6: 13–16, 18
4	9.B+	applied square roots	1–6		9.10: 20–21, 25–32
1	10.1+	linear modeling, constraints, simultaneous graphs	1–8		10.1: 9–11, 13–17
2	10.2+	mixture	1–10M		10.1: 18–22
					P 10.2: 1–5
3	10.3+	simultaneous equations with Lab Gear	1–11		10.3: 16–27
4	10.4+	solving simultaneous equations by substitution	1–10		P 10.4: 1–6
					10.5: 1–5
5	10.5+	Ax + By = C	6, 14, 15, 20, 21	G: 14–21	10.5: 22
					10.6: 7, 8
6	10.6+	simultaneous equations and graphs	1–6, 9–12	G: 21–26	10.6: 15–18, 28–30
7	10.6+	simultaneous equations and graphs	20		Finish classwork.
	10.7+	two-variable word problems	1–6, 9, 13, 14		
8	10.8+	equations of lines	1–12, 15–17		P 10.8: 1–6
9	10.B+	fitting a line	1–6		10.B: 7

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Day	Lesson	Topic	Classwork	Notes	Homework
1	11.3✦	slope	1–8		11.5: 1
2	11.5✦	introduction to probability	2–7, 9–12M		11.5: 15–19
3	11.6✦	more probability	1–5, 11–15		11.5: 20–22 11.6: 6–10
4	11.8	unit conversion	1–12		11.8: 19–23 p.423: 19–22b
1	12.8✦	linear graphs and review of function diagrams	1–10	11–21: Nice for enrichment.	Finish classwork.
1	13.6✦	completing squares	1–4, 8–14, 23		13.6: 15–22, 26, 27
1	14.2✦	rational expressions	1–7, 9–13		14.2: 24–26
2	14.3✦	rational expressions	1–13		p.507: 3–8, 10–17

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Alternate Paths: The Graphing Path

The graphing path takes fullest advantage of graphing calculators or computers with graphing capability in the classroom. It adds to the basic-plus path:

- Investigations on the relationship between the equation of a function and its graph, including translation of graphs and the effect of degree on graphs of polynomials.

- Extensive work on graphing quadratic functions.

- Graphic solutions to optimization problems.

In order to complete the graphing path in 130 class days, the following topics are deleted from the basic-plus path:

- Geoboard investigations of area and similarity.

- Differences of squares and perfect square identities.

- Application of a square root function.

- Modeling with linear systems.

- Probability.

- Extended work with slope.

ADD THESE LESSONS AND THE GRAPHING CALCULATOR PROBLEMS (MARKED "G") TO THE BASIC-PLUS PATH:	DELETE THESE LESSONS FROM THE BASIC-PLUS PATH:
5.5 *M*	1.12
7.B	2.12
12.A	3.12
12.B	7.4 (except "Review")
13.1	7.5
13.2	8.6
13.3	9.B
13.4	10.1
13.5	10.2
13.7	10.7
13.8	11.3
14.4	11.5
14.6	11.6

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Alternate Paths: The Applications Path

The applications path focuses on relating algebra to the real world. It adds to the basic-plus path:

- Collecting measurements and dealing with measurement error.
- Fitting a line to data.
- More work on exponential growth, including compound interest, and geometric mean.
- More probability.
- More unit conversions.
- Modeling population growth and motion.
- Applications from science: linear dependence, iterating linear functions, and direct, inverse, and combined variation.
- Optimization.

ADD THESE LESSONS TO THE BASIC-PLUS PATH:

4.7
4.8 (2 days) *M*
4.9
8.6: "population predictions"
8.B
9.7 (2 days)
11.7
11.B
12.1
12.2 *M*
12.3
12.4
12.A
12.5
12.7 (2 days)
13.1: 1-10
13.4
13.A
13.5
14.A

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In order to complete the applications path in 130 class days, the following topics are deleted from the basic-plus path:

- Equivalent fractions and similarity.
- Constant sums and products.
- Factoring quadratic polynomials, including differences of squares and perfect squares.
- Extended work on properties of exponents, linear graphs and function diagrams, and rational expressions.
- Solving linear systems.

DELETE THESE LESSONS FROM THE BASIC-PLUS PATH:

3.12
5.1
5.2
5.4
5.6
7.1
7.2
7.3
7.4 (except "Review")
7.5
8.10
10.2
10.3
10.5
10.6
10.7
12.8
13.6
14.2

Additional Strands

Use the strands described below to create a core path tailored to your needs and interests. Start with the basic-plus, graphing, or applications path and substitute the desired lessons for those which you find less compelling. (It is not recommended to remove lessons that are part of the basic path, however.) You may also have time to explore one or more of the additional strands after completing your core path. You can easily go back in the text to pick up lessons you skipped earlier.

Strand	Lessons in Basic-Plus Path	Additional Lessons
Abstract algebra: identity and inverse elements in finite groups, distributive laws.		3.11, 4.C, 5.B, 5.12, 6.C
Exponential growth: extended work, including negative exponents, compound interest, population growth, and geometric mean.	8.6	8.B, 9.7, 9.8, 12.1
Factoring: more complicated quadratics.	5.4, 5.6 7.4, 7.5: 11-13	2.4:"Related Products", 5.3: "Multiplying Binomials," 5.A, 5.7, 6.6: 28-33, 7.3: "Recognizing Perfect Squares," 7.5
Geometry: perimeter, area, volume, dimension, similarity, golden ratio.	3.12	1.10, 2.10M, 6.11, 7.A, 9.10, 9.11M, 9.12, 9.C, 14.1, 14.8
Probability: outcomes and events, relative frequency, theoretical probability.	11.5M, 11.6	11.7
Quadratic equations and functions: vertex form, intercept form, solving by graphing, equal squares, factoring, quadratic formula.	7.6, 7.7	5.5M, 13.1, 13.2, 13.3, 13.6, 13.7, 13.8, 13.B, 14.4, 14.5, 14.6, 14.7
Sequences: Fibonacci, arithmetic and geometric.		2.6: "Fibonacci Sequences," 2.10M, 5.9, 5.10, 5.11, 5.C, 11.1, 11.2, 14.8
Square roots: extended work with simplifying radical expressions, rational and irrational numbers.	9.B	9.3, 9.4, 9.A, 9.5, 9.8, 9.9, 9.C, 10.B "Discovery", 11.2, 11.4, 11.A