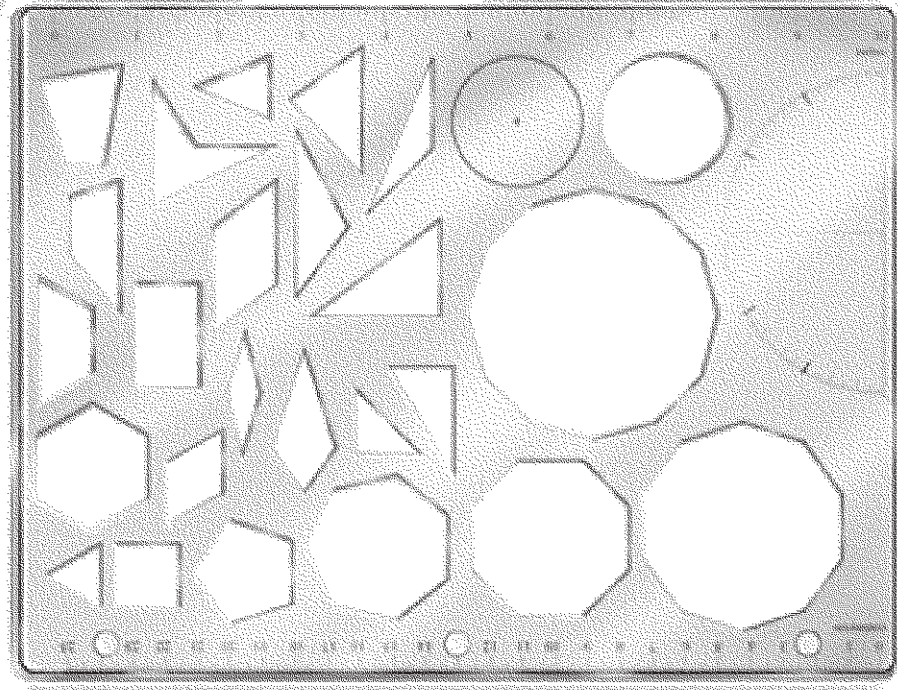


6 Triangles and Quadrilaterals



The content of this section is close to some of the topics covered in a traditional high school geometry course. The approach, however, is quite different from that which is followed in most textbooks. The labs work well as discussion starters, and they can be used as supplements to the traditional curriculum before, during, or after the corresponding work on triangles and quadrilaterals.

A note on terminology: I use *equal* when talking about sides or angles and *congruent* when talking about polygons. The distinction between similar and congruent polygons is important to make at this level, and the terminology is helpful there. However, the distinction between congruent sides and equal measurements is rather subtle and really not worth the trouble unless one is developing a very formal axiomatic system. Such formality is often purchased at the price of losing many students and, in fact, the geometric content of the course, as one gets mired in terminology and notation.

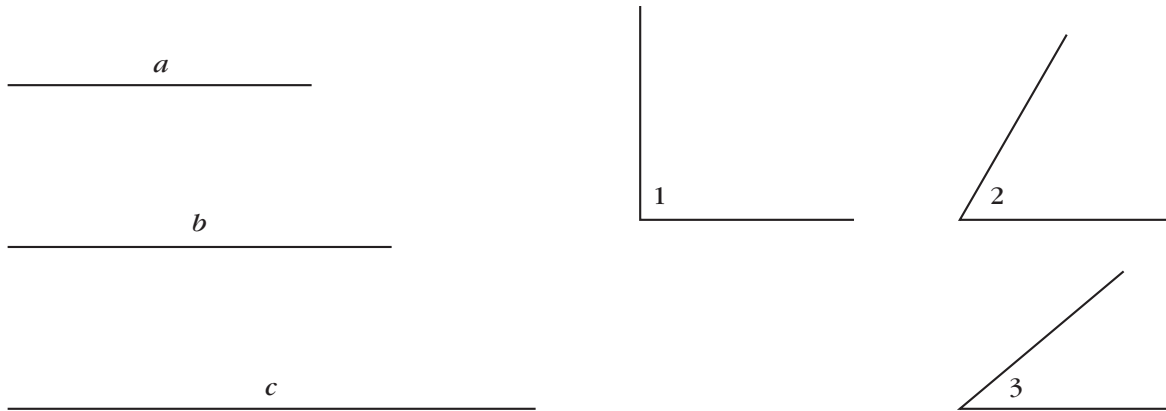
See page 199 for teacher notes to this section.

LAB 6.1

Noncongruent Triangles

Name(s) _____

■ **Equipment:** Compass, straightedge, unlined paper



Instructions: For the following problems,

- a. Use the compass and straightedge to construct *two noncongruent triangles* that satisfy the given conditions.
 - b. Label the sides and angles that are equal to those shown above.
 - c. If you cannot construct two noncongruent triangles, explain why, and construct just one triangle.
 - d. If you cannot construct even one, explain why.
1. One side of length a and an adjacent angle equal to $\angle 3$
 2. One side of length a and an opposite angle equal to $\angle 3$
 3. One angle equal to $\angle 1$ and one equal to $\angle 2$
 4. One angle equal to $\angle 1$, one equal to $\angle 2$, and one equal to $\angle 3$
 5. One side of length a , one of length b , one of length c
 6. One side of length a , one of length b , and between them an angle equal to $\angle 3$
 7. One side of length a , one of length b , and an angle equal to $\angle 3$ opposite the first side
 8. One side of length a between angles equal to $\angle 2$ and $\angle 3$
 9. One side of length a adjacent to an angle equal to $\angle 2$ and opposite an angle equal to $\angle 3$
 10. One side of length a , one of length b , one of length c , and an angle equal to $\angle 1$

LAB 6.1

Name(s) _____

Noncongruent Triangles (continued)

Discussion

- A. Problem 1 could be described as an SA construction (one pair of equal sides and one pair of equal angles). Problem 6 could be described as an SAS construction (two pairs of equal sides with one pair of equal angles between them). Problem 7 could be described as an SSA construction (two pairs of equal sides with one pair of equal angles not between them). Make a two-column table to summarize your work in Problems 1–10.
- In column 1, describe the problem using the letters S and A; each S represents a pair of equal sides, and each A represents a pair of equal angles.
 - In column 2, explain whether the given conditions lead to many possible triangles, two possible triangles, one possible triangle, or no triangles.
- B. Which problems had no solution? Why?
- C. Which problems had a unique solution? How can this help us recognize congruent triangles?
- D. Which problem had exactly two solutions?

LAB 6.2

Name(s) _____

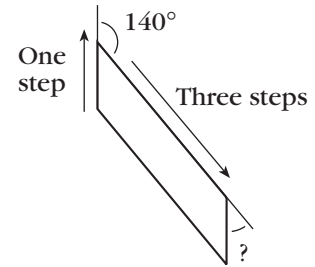
Walking Parallelograms

The figure at right shows the beginning of a parallelogram walk:

Walk forward one step;

Turn right 140° ;

Walk forward three steps.



1. The third side must be parallel to the first. What must the total turning be for the first two turns?
2. Complete the instructions for walking this parallelogram so the walker ends up in the same place and position where he or she started.
3. What are the interior and exterior angles for this parallelogram?
4. Suppose a parallelogram has sides of length x and y and one interior angle measuring a° . Write instructions for walking it.
5. Use variables in the same way to write instructions for walking the following figures.
 - a. A rhombus
 - b. A rectangle
 - c. A square
6. Given any rhombus, can you choose values for the variables in the parallelogram instructions so that someone following them would walk that rhombus? Explain.
7. Given any parallelogram, can you choose values for the variables in the rhombus instructions so that someone following them would walk that parallelogram? Explain.

LAB 6.2

Name(s) _____

Walking Parallelograms (continued)

8. Use the answers to Problems 6 and 7 to answer these questions.
- a. Is a rhombus a parallelogram?
 - b. Is a parallelogram a rhombus?
9. Repeat Problems 6 and 7 for other pairs of quadrilaterals among these four: parallelogram, rhombus, rectangle, square. There are four possibilities besides the rhombus–parallelogram pair.
10. Use the answer to Problem 9 to decide which of the four quadrilaterals are specific examples of one of the others.

Discussion

- A. What is the relationship between consecutive angles of a parallelogram?
Between interior and turn angles?
- B. How many variables does each of your sets of instructions in Problems 4 and 5 have? What does this tell you about the corresponding shape?
- C. What values are allowed for each variable? What values are not?
- D. For which two of the shapes (parallelogram, rhombus, rectangle, square) is neither one an example of the other?
- E. How would you define congruence rules for each of these four quadrilaterals?
For example, “Two squares are congruent if they have a pair of equal sides (S).”
- F. Some math teachers believe that a trapezoid should be defined as a quadrilateral with *exactly* one pair of parallel sides. Others believe that a trapezoid should be defined as a quadrilateral with *at least* one pair of parallel sides. How are these choices different? How do they affect the classification of quadrilaterals? Whom do you agree with and why?

LAB 6.3

Name(s) _____

Making Quadrilaterals from the Inside Out

■ **Equipment:** Dot or graph paper, straightedge

Definition: To bisect means to cut *exactly* in half.

Instructions: For the following problems,

- a. On dot or graph paper, draw two intersecting line segments that satisfy the given conditions. These will be the diagonals.
- b. Join the endpoints to make a quadrilateral.
- c. Name the quadrilateral.

Whenever possible, try to make a quadrilateral that is not a parallelogram, a trapezoid, or a kite.

1. Perpendicular, equal segments that bisect each other
2. Perpendicular, equal segments that do not bisect each other
3. Perpendicular, unequal segments that bisect each other
4. Perpendicular, unequal segments that do not bisect each other
5. Nonperpendicular, equal segments that bisect each other
6. Nonperpendicular, equal segments that do not bisect each other
7. Nonperpendicular, unequal segments that bisect each other
8. Nonperpendicular, unequal segments that do not bisect each other

Discussion

- A. For four of the Problems 1–8, it is possible to get a quadrilateral that is not a parallelogram, a trapezoid, or a kite. What do they all have in common?
- B. What requirements do you need to put on the diagonals to get a kite? (It is not one of conditions 1–8.)
- C. What requirements do you need to put on the diagonals to get an isosceles trapezoid? (It is not one of conditions 1–8.)

LAB 6.4

Name(s) _____

Making Quadrilaterals from Triangles

■ **Equipment:** Template

Eight types of triangles

Equilateral (EQ)

Acute isosceles (AI)

Right isosceles (RI)

Obtuse isosceles (OI)

Acute scalene (AS)

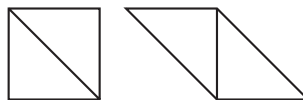
Right scalene (RS)

Half-equilateral (HE)

Obtuse scalene (OS)

Use a separate sheet or sheets of paper to prepare a well-organized and clearly illustrated report that answers the following questions.

1. Using two congruent triangles placed edge to edge, what types of quadrilaterals can you make? For example, as shown below, with two congruent right isosceles triangles, you can make a square or a parallelogram. Discuss all eight types of triangles.



2. Using three congruent triangles placed edge to edge, what types of quadrilaterals can you make?

LAB 6.5

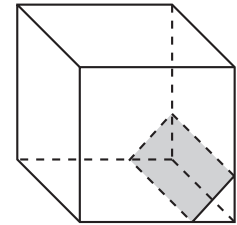
Slicing a Cube

Name(s) _____

■ **Equipment:** Transparency, stiff paper, scissors, adhesive tape

Imagine that you slice a cube in a direction parallel to one of the faces. The shape of the slice will be a square. Now imagine that you slice a cube parallel to one of the bottom edges, but tilted from the horizontal. The shape of the slice will be a rectangle.

What shapes are possible for a slice? To investigate this, you will use a hollow transparent cube and stiff paper to simulate the slices.



1. Cut five squares out of the transparency, each one with sides 3 inches in length. Tape them together to make a lidless cubic box.
2. Cut a 1- by 3-inch rectangle out of stiff paper, and place it in the cube so as to match the slice shown in the figure above.
3. Cut other shapes out of stiff paper to show slices that match each of the following polygons, if possible. Sketch your solutions on a separate sheet of paper.

Triangles:

Equilateral (EQ)
Acute isosceles (AI)
Right isosceles (RI)
Obtuse isosceles (OI)
Acute scalene (AS)
Right scalene (RS)
Half-equilateral (HE)
Obtuse scalene (OS)

Quadrilaterals:

Square (SQ)
Rhombus (RH)
Rectangle (RE)
Parallelogram (PA)
Kite (KI)
Isosceles trapezoid (IT)
General trapezoid (GT)
General quadrilateral (GQ)

Other polygons:

Pentagon
Regular pentagon
Hexagon
Regular hexagon
Seven-gon

Discussion

- A. What polygon slices are possible? Impossible? Explain.
- B. What is the range of possible sides and angles for each polygon slice?
- C. What is the most symmetric triangle slice? Quadrilateral? Pentagon? Hexagon?
- D. Is there a square slice that is not parallel to one of the faces?