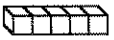
The Algebra Lab Gear™

These are the blocks that make up a set of Algebra Lab Gear.

1



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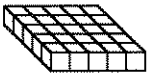


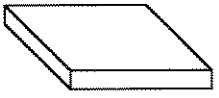
25

x

x^2

$5x$

y



y^2

$5y$

xy

Chapter 1 Meeting the Algebra Lab Gear™

This chapter introduces all of the Lab Gear, with the exception of the corner piece.

New Words and Concepts

This chapter does not assume an in-depth understanding of signed number arithmetic, which is introduced in Chapter 2, but it does introduce several important concepts of algebra, which will be returned to throughout the book. These include:

variables and **constants**
combining like terms
the various meanings of the **minus** sign
squaring a quantity
cancelling

Teaching Tips

If your class has had experience with basic algebra, you may want to speed through this chapter. However, do not skip too many exercises or too much reading, since your students need to master the Lab Gear vocabulary, and the basic Lab Gear techniques, before going on. In particular, make sure they know the name of each of the blocks, and that they understand the two ways to illustrate the minus sign and its three meanings. In any case, it is recommended that you do not skip the Explorations. (For notes on the Explorations, see page ix.)

The corner piece will make its first appearance in Chapter 3. If your students are curious about its purpose, encourage them to guess.

Lesson Notes

- **Lesson 1,** The Blocks, page 3: Show the students how to use the x- and y-blocks to measure the sides of the rectangles in order to identify the x^2-block, the y^2-block, and the xy-block.
- **Lesson 2,** Sketching the Lab Gear, page 5: Discourage perfectionism here. The sketches should allow one to recognize the blocks, but they need not be works of art.
- **Lesson 3,** Variables, page 6: This is a crucial activity to establish the concept of variable. Note that no negative numbers are used, yet.
- **Lesson 4,** Like Terms, page 7: This lesson should help prevent mistakes of the type $x + x = x^2$. If students want to do this lesson by looking at the figures, without using actual blocks, let them, unless they still have trouble recognizing the blocks. At this stage, do not use abstract arguments such as the distributive rule to explain combining like terms—the blocks make it clear enough.
- **Lesson 5,** Minus, page 9: Misunderstandings about the meaning of the minus sign are a major obstacle to students' understanding of algebra. It is best to start talking about this confusing but powerful symbol explicitly early on. You may want to lead a class discussion of the ideas in this lesson.
- **Lesson 6,** Minus with the Lab Gear, page 10: Be sure to make copies of the workmat (page 131) for all your students. If they keep their work in a binder, make sure that the workmat has holes punched in it.

  The Lab Gear does not use color to differentiate positive from negative numbers, because that approach cannot be generalized to variables. ($-x$ is not necessarily negative.) The two methods presented in this lesson, used in combination, make it possible to have a unified manipulative environment for algebra. Using color, or using just one of the methods at a time puts enormous limits on the flexibility of an algebra manipulative program.
- **Lesson 7,** Opposites, page 12: Stress that the same number can be represented in many ways. Cancelling and adding zero ("uncancelling") both turn out to be critical skills in algebra and in manipulating the Lab Gear.
- **Lesson 8,** More on Minus, page 13: Notice that no attempt is made here to teach or review signed number arithmetic. Instead, students are shown how to simplify expressions that involve the minus sign by manipulation of the Lab Gear.

The Blocks

Look at your Lab Gear blocks. There are two kinds of blocks, yellow and blue.

The Yellow Blocks

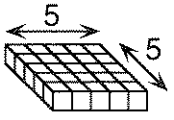
The yellow blocks represent whole numbers, such as 1 or 5.

Find blocks in your Lab Gear that you can use to show these quantities.

1. 3
2. 13
3. 31
4. Write some numbers that *cannot* be represented by the Lab Gear. Discuss your answers with your classmates.

You will soon learn to use the Lab Gear for negative numbers. Later, you will use the Lab Gear to work with fractions.

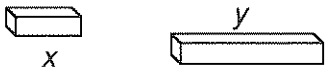
Notice that the block that represents 25 is a 5 by 5 square. In algebra, the multiplication 5 times 5 is written $5 \cdot 5 = 25$, or $5\,(5) = 25$. Do not use $\times$ to indicate multiplication—it could be confused with the letter *x*.



5. Use the blocks to show 30 as a rectangle. Find four different ways. For each way, write a multiplication equation.

The Blue Blocks

The blue blocks represent *variable quantities*. All of them are related to these two blocks.



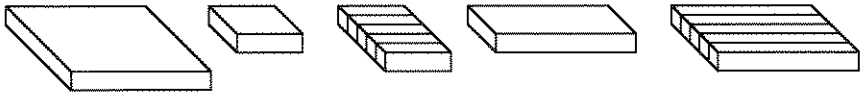
Variables are usually named by letters. Since the names x and y are used most often in algebra books, they have been chosen to name the variables in the Lab Gear.

6. Write a way to remember which block is x, and which is y.

In algebra, 5 times x is usually written $5x$. (You do not need to write the multiplication dot between a number and a variable, or between two variables.)

7. Find blocks that show these quantities.
   a. 5 times x (write $5x$)
   b. 5 times y
   c. x times y (write xy)
   d. x times x (write x^2)
   e. y times y

   Label the blocks in this figure.



The expression x^2 is read x **squared** or *x to the power 2*. It means *x multiplied by itself*. In algebra, x and y are called variables because they can stand for different numbers at different times.

Copy and complete these sentences.

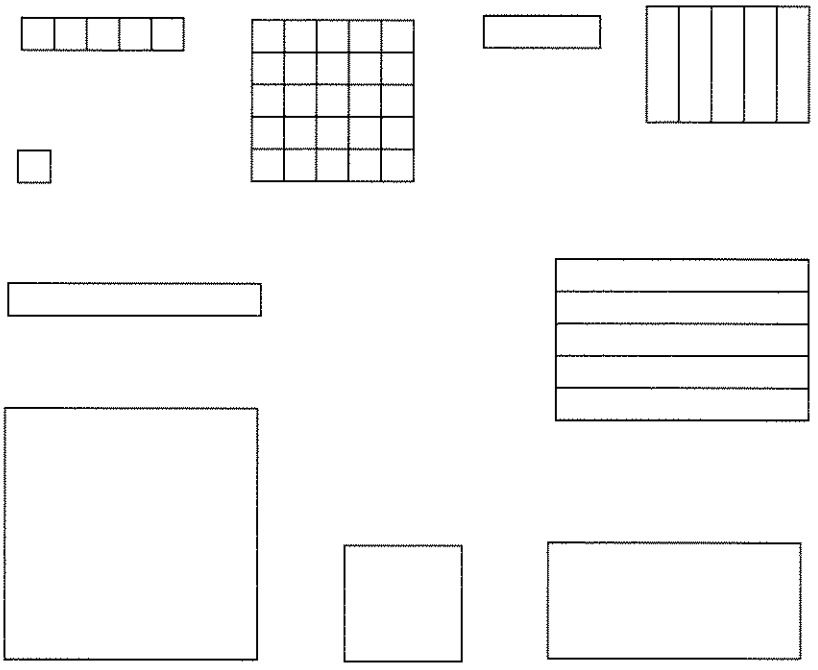
8. If $x = 6$, then $5x =$ _____, and $x^2 =$ _____.
9. But if $x = 6.2$, then $5x =$ _____, and $x^2 =$ _____.

We agree that the x-block and the y-block can represent any numbers, including negative numbers and fractions. Even though the x-block is shorter than the y-block, it could represent a greater number. Or x could be equal to y, or less than y.

10. Explain why x^2 is read *x squared*.

Sketching the Lab Gear

You will often be asked to sketch solutions to problems. It is easier to sketch the Lab Gear blocks two-dimensionally, as they appear when seen from above.

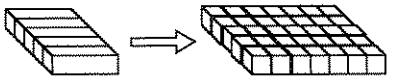


1. Label each of these block sketches.
2. Put a check next to the ones that are **constants** (not variables).
3. On separate paper, practice sketching each of the blocks.
4. Next to each of your sketches, write what the value of the block would be if $x = 3$ and $y = 2$.
5. Draw three-dimensional pictures of some of the Lab Gear blocks.

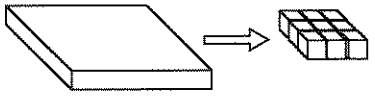
Variables

The blue blocks represent variables. They can stand for different numbers. When you know the value of the variable you can replace the blue blocks with the appropriate yellow blocks. Look at these examples.

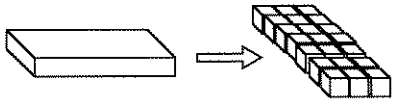
Here is $5x$ if $x = 7$.



Here is y^2 if $y = 3$.

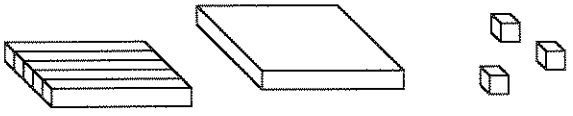


Here is xy if $x = 7$ and $y = 3$.



To answer these questions, first put out blocks to match each figure. Then, when possible, replace the variables with the given constants, and count what you have.

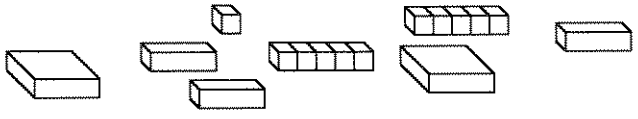
1.



Write what the blocks show if:

a. $y = 1$
b. $y = 2$
c. $y = 0$
d. $y = 1.3$

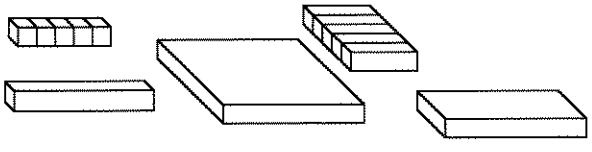
2.



Write what the blocks show if:

a. $x = 2$
b. $x = 1$
c. $x = \frac{1}{2}$
d. $x = 1.5$

3.



Write what the blocks show if:

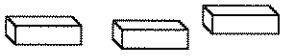
a. $x = 5$ and $y = 4$
b. $x = 3$ and $y = 1$
c. $x = 1.2$ and $y = 2$
d. $x = \frac{1}{2}$ and $y = 4$

4. Make up a problem of this type for another student.

Like Terms

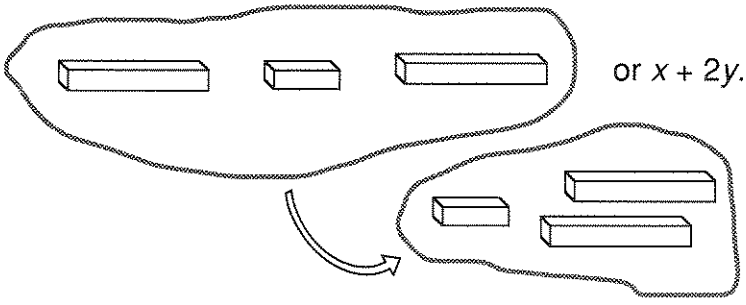
You can write an algebraic expression that names a group of blocks many ways. If you group the blocks together that have the same size and shape, that is a way to show **combining like terms**. Look at these examples.

This quantity is written $x + x + x$,



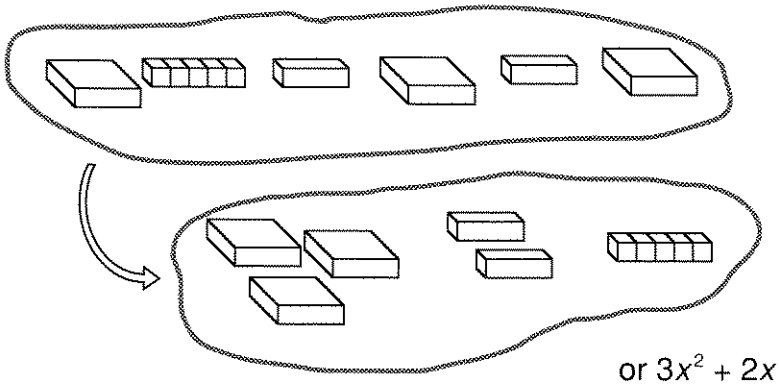
or $3x$.

This quantity is written $y + x + y$,



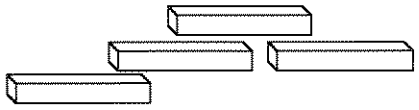
or $x + 2y$.

This quantity is written $x^2 + 5 + x + x^2 + x + x^2$,



or $3x^2 + 2x + 5$.

For each example, show the figure with your blocks, combine like terms, then write the quantity the short way.

1. 

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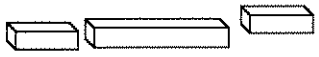
3. 

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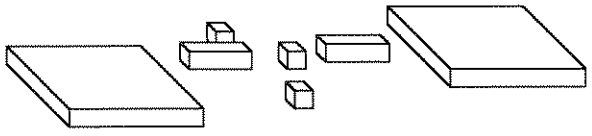
5. 

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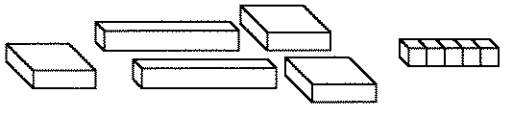
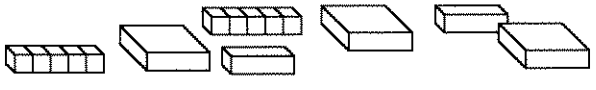
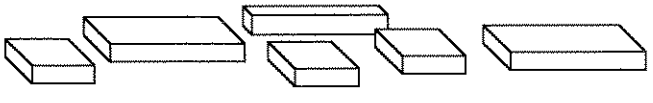
7. 

8. 

9. 

Minus

The **minus** sign can mean three different things, depending on the context.

- In front of a number, and only there, it means *negative*.
  For example, –2 can mean negative 2.
- In front of any expression, it means *the opposite of*.
  For example, –2 can mean the opposite of 2, which is negative 2.

  Also, –(–2) means the opposite of –2, which is 2. (Notice that –(–2) could be written – –2. The parentheses are added to make it easier to read.)

  For another example, $-x$ means the opposite of x. So, if x is positive, $-x$ is negative. If x is negative, $-x$ is positive.

  Also, $-(2x + 1)$ means the opposite of $2x + 1$. So, if $2x + 1$ is positive, $-(2x + 1)$ is negative. If $2x + 1$ is negative, $-(2x + 1)$ is positive.
- Between two expressions, it means *subtract the second expression from the first one.*

  For example, $x - 3$ means subtract 3 from x. Also, $3 - x$ means subtract x from 3.

For each expression, write an explanation of what the minus signs mean.

1. -3
2. $5 - 2$
3. $-(-5)$
4. $3 - 7$
5. $-(4 + \frac{1}{2})$
6. $-(2 - 6)$
7. $3 - x$
8. $-(-x)$
9. $2y - y$
10. $-(4x + 1)$
11. $y - 5$
12. $y - (-x)$

Exploration 1 Positive or Negative?

1. Write the value of $-x$ if:
   a. $x = 2$
   b. $x = -3$
2. True or False?
   a. $-x$ is always negative
   b. $-x$ can be positive

   Explain your answers.

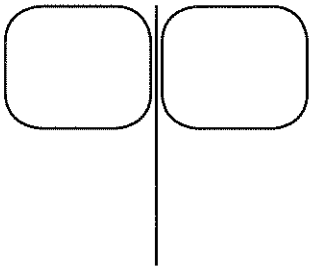
Minus with the Lab Gear

With the Lab Gear, we show the minus sign in two different ways.

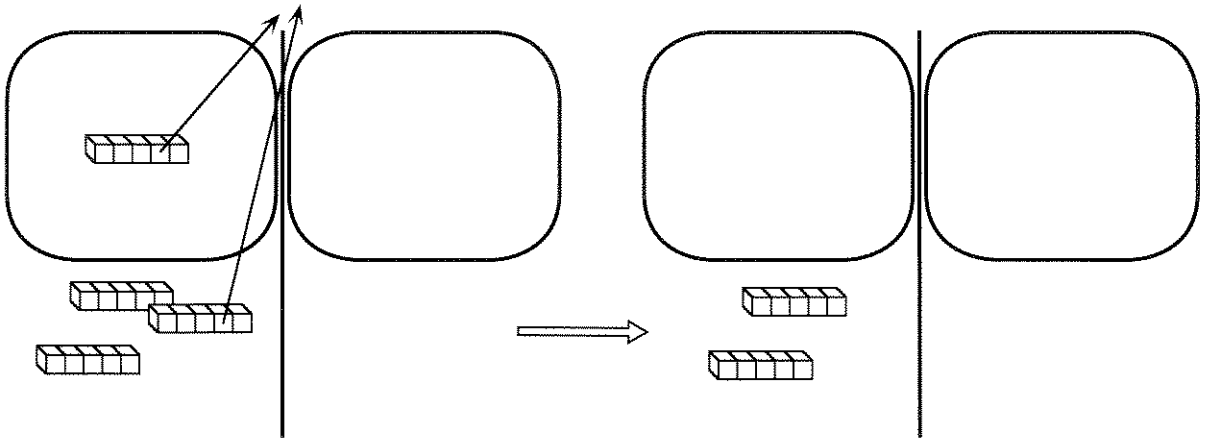
- We can put blocks inside the *minus area* of the workmat.
- We can put blocks *upstairs* (on top of other blocks).

The Minus Area

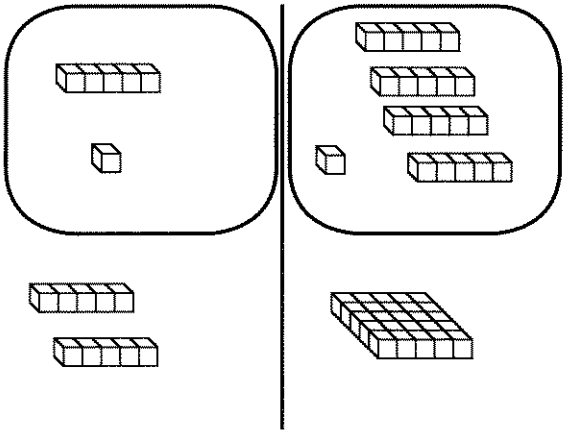
Look at your workmat. The rectangles with rounded corners represent the *minus areas*. If you remove matching blocks from inside and outside the minus area, the remaining blocks show the simplest way to write the expression.



For example, the subtraction equation 15 – 5 = 10 can be shown this way on the workmat.



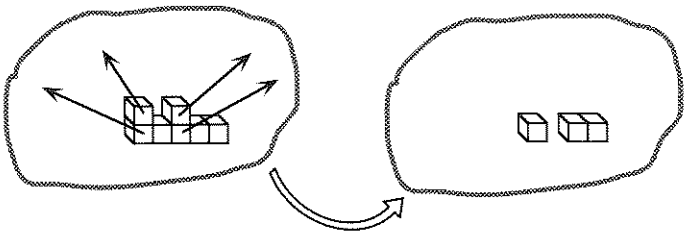
The problem above was worked on one side of the line on the workmat. When using both sides, the line usually represents an equal sign. For example, this workmat shows the equality 10 – 6 = 25 – 21.



This is an equality since both sides, once simplified, represent the same number.

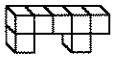
1. Mark the blocks that can be removed. Write what both sides show when simplified by removing blocks.

Upstairs



This figure shows 5 – 2. Notice that the *uncovered* part of the bottom block equals 3. If you remove matching upstairs and downstairs blocks, you will be left with three downstairs blocks. This is how we show 5 – 2 = 3 with upstairs and downstairs blocks.

This figure shows 2 – 5. If you mentally remove matching blocks downstairs and upstairs, you are left with 3 upstairs blocks, or –3. We can only do this mentally, however, since blocks cannot float in mid-air.



2 – 5 = –3

For each figure, write a subtraction expression.

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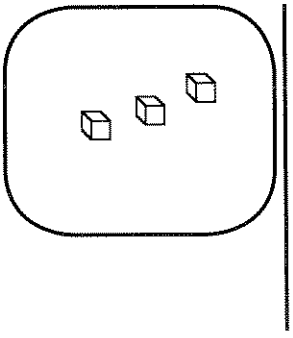

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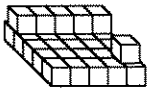

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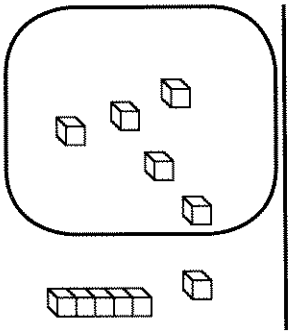

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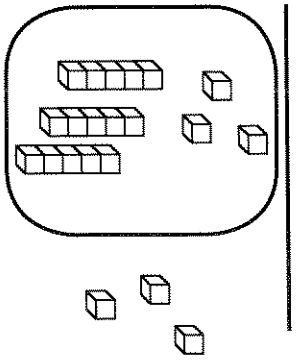

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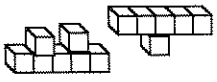

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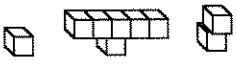

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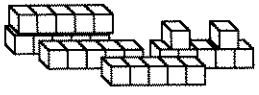

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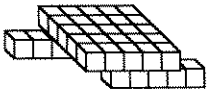
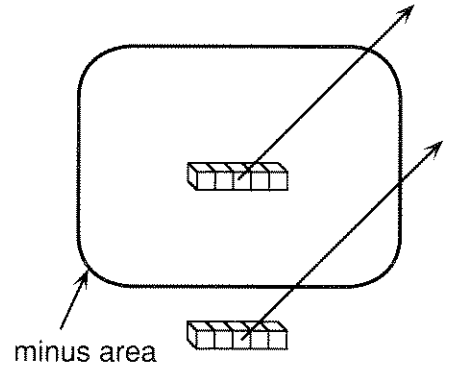


Opposites

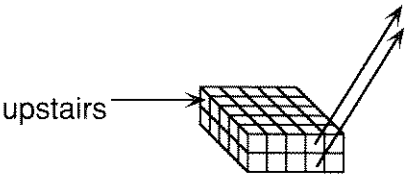
You will use the workmat to work with **integers**. Integers are positive and negative whole numbers, and zero. When you add opposites, such as 3 and –3, you get zero. When modeling algebraic expressions on the workmat, we can use informal language and say that 3 and –3 can be **cancelled**. Look at these two examples of cancelling.

This example of cancelling uses the minus area.



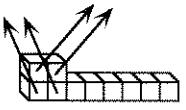
5 + (–5) = 0

This example of cancelling shows blocks *upstairs*.



25 + (–25) = 0

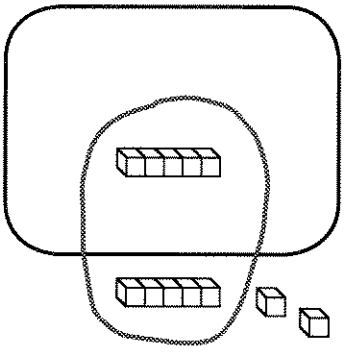
Usually, after cancelling, something is left. This example shows what happens if we start with 8 downstairs, and 2 upstairs. After cancelling, 6 blocks are left downstairs. We say that 8 downstairs and 2 upstairs is a way of showing the number 6.



1. a. What is left after cancelling?

   b. What number do 9 blocks in the minus area, and 2 blocks outside show?

Sometimes, it is useful to show a number with more blocks. For example, the number 2 can be shown with two one-blocks outside the minus area. But even after adding a five-block in the minus area and a five-block outside the figure still shows 2.

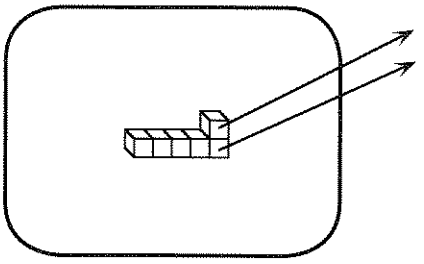


2. Sketch two other ways to show the number 2.

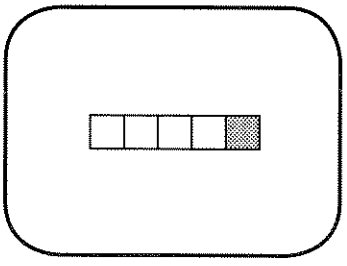
More on Minus

Minus signs can be a source of many errors for algebra students, but using the Lab Gear makes it easier to think about minus. Consider what happens when we have upstairs blocks inside the minus area.

For example, in this figure, we have the quantity (5 – 1) inside the minus area. This is written –(5 – 1). Since the upstairs and downstairs blocks cancel, the quantity simplifies to –4.



For a quick sketch of this, draw the 5-block as it is seen from above, and shade in the part that is covered by the 1-block.

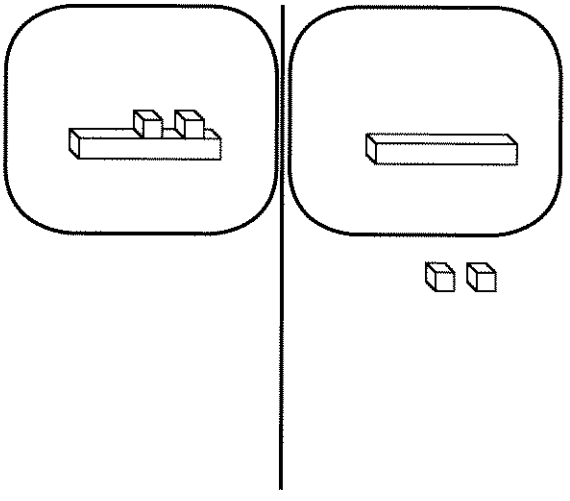


But this sketching method does not work if the upstairs block is bigger than the downstairs block. In that case, try for a three-dimensional picture.

1. Find two other ways to show –4 using only a 5-block and a 1-block. Sketch your solutions.

Do not stack Lab Gear blocks more than two levels high. Two levels is enough to illustrate many ideas of algebra, and will keep things clear. More would be confusing.

Now consider this figure. The left side shows the opposite of $(y - 2)$, We write, $-(y - 2)$. The right side shows the opposite of y, and 2. We write, $-y + 2$. The vertical line means =. So to record the figure, we can write, $-(y - 2) = -y + 2$.



2. Do you think this statement is always true? Explain your answer. (Try different values for y, such as –5, –1, 0, and 2. See if they make the statement true.)

3. a. Sketch a figure to illustrate the statement: $-(5 - x) = -5 + x$
   b. Do you think this statement is always true? Try different values for x to test it, such as –7, –3, 0, 2, and 8.
4. Look at the figure for problem 2, and at the figure you drew for problem 3. What can you say about upstairs blocks in the minus area?

Exploration 2 Minus Puzzles

Use your Lab Gear to solve these puzzles.

1. Find four ways to show 3 with three blocks.
2. Find four ways to show –8 with four blocks.
3. Show –9 with three blocks.
4. Show –9 with five blocks.
5. Show –9 with seven blocks.