# Scientific Notation Teacher Notes

This unit is adapted from *Algebra: Themes, Tools, Concepts* by Anita Wah and Henri Picciotto, Lessons 7.9 to 7.11. The book is available for free download at
www.MathEducationPage.org/attc/
Also available on the Web site: a Teachers' Edition with answers and additional notes, as well as some quizzes and tests.

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## Introduction

None of the work in this unit can be done without scientific or graphing calculators. Scientific notation is mostly used in contexts like science and engineering where the use of electronic tools is taken for granted. Moreover, the Common Core recommends appropriate use of tools at all times. Finally, and perhaps most importantly, access to calculators helps students whose arithmetic skills are weak reengage with mathematics.

The main purpose of the unit is to teach the mathematics of scientific notation:
- The meaning of exponentiation (raising to a power)
- How exponentiation makes it possible to express large numbers in a compact way
- How a combination of multiplication and exponents makes it possible to approximate large numbers
- How scientific notation makes it possible to compare magnitudes at a glance
- How scientific notation makes it possible to discuss real-world problems that involve very large numbers
- How to convert numbers to and from scientific notation
- How to multiply and divide numbers in scientific notation.

It is important to keep in mind that the final two items on this list are pointless if the earlier items are not understood. Not only that, but for many students they would be very difficult or impossible to remember those skills in the absence of that understanding.

## 1. Approximating Large Numbers

Before diving into the mechanics of how to convert numbers to and from scientific notation, it is useful to lay conceptual groundwork by working with exponents other than ten. They are not as easy to work with, and require a lot of trial and error on the calculator. This is a good thing, for several reasons:
- For most adults, trial and error on the calculator is by far a more useful skill than scientific notation
- Experimenting like this makes it easier to understand what is going on with scientific notation
- Scientific notation is much easier than working with powers of numbers other than 10, and students will appreciate that!

Before they can work on #1, students need to know the meaning of exponents. You might review this not by an explanation, which students may not hear, but with some mental arithmetic exercises, such as $2^4$, $3^2$, and $10^3$. Ask students to write down their answers to these problems, without using their calculators or doing the calculations on paper. Have students share how they get their answers.

They will also need to know how to use their calculators to calculate powers. For that you may ask them to use their calculators to find numbers like $9^4$ and $654^3$. It is useful to have students answer this by repeated multiplication, and by using the power key on the calculator, as it reinforces the meaning of exponentiation.

After students have worked on #1, have a whole-class discussion of the answers. This would help students get ready to tackle #2-4.

#5-9 are intended to lay some groundwork for the rest of the unit.

For #5, do not be too demanding, as students probably don't have a good sense of what makes for a good estimate. Instead, collect student answers to one of #2-4, and point out the powers of 2, 3, 9 and 10 are not that close to the desired number. Tell students they will learn a way to make much better estimates with the help of exponents later in the unit.

#6-9 are a bit of a prerequisite to further work.

Instead of spending a lot of time doing examples, insist on students putting in words the strategies for #6-8, and have them several students read their answers to the class. Then lead a respectful discussion of those strategies.

#6-8 should lead to an answer for #9. If no student comes up with an answer, you can ask for guesses, and you can offer hints. If someone has an idea that involves counting digits, have everyone check whether it works, but don't suggest it yourself, as this is the sort of thing that is hard to remember. If no one comes up with a strategy, you should suggest an answer based on asking "which powers of 10 is this number between?"

## 2. Closer Approximations

This lesson involves more calculator trial and error. It is essentially about a sort of scientific notation using bases other than 10. (Do not use the words "scientific notation" yet!)

Ask the students to read the introduction to the lesson, and to explain it to their neighbors. Then, lead a whole class discussion leading to #1, which should also be discussed as a whole class before going on.

After that, students should be ready to do #2-5.For #5, hopefully many students will choose 10 as a base!

At the end of the lesson, ask students to vote on which base is the most convenient. Since they will almost certainly choose 10, you can congratulate them, and tell them that they agree with mathematicians and scientists. At this point you can tell them that using 10 as a base and multiplying by a number between 1 and 10 is called *scientific notation* and that they will learn more about it in future lessons.

## 3. Scientific Notation

The definition of scientific notation is based on the ideas developed in the previous lessons.

For #5, you will need to give students access to the needed resources. If that is inconvenient, it is still worth doing, because students will have more invested in doing exercises based on numbers they found, and it makes the point about the relevance of scientific notation better than any teacher speech. If it is impossible to give them access to such resources in class, this assignment may work as homework. If that too is impossible, you might do some Web searching on your laptop, based on student suggestions.

If students find numbers that are already given in scientific notation, they can be asked to translate them into normal decimal notation.

### 4. Scientific Notation on a Calculator

This lesson may need to be adapted to whatever calculator your students have access to.

Note that the questions go back and forth between questions about numbers and questions about calculators. This is intentional, as there is no reason to separate the two. If a student picks up the calculator instructions quickly, they still have some math to think about, and vice versa.

#7 is essential!

### 5. Look, Up in the Sky!

This is an application of scientific notation to astronomy.

#1: The two groups are objects in the solar system vs. stars.

In answering #2-4, students will find that answering (a) (how much further? in other words a subtraction) is not very helpful. (Try it!) The reason is that if the larger number is much larger, subtracting the smaller number does not have much of an effect on the size of the answer.

Answering (b) (how many times as far? in other words a division) is much more useful when numbers are large and far apart.

### 6. Without a Calculator

#2: #1b could be figured out by using the distributive law.

#4-7: It is crucial to have a full discussion of these exercises. Tell students that they will have to present their understanding of these ideas in the next lesson. Solicit their methods for finding the answer without a calculator. If students have different answers, have them discuss which answers make the most sense to them, based on the explanations of how the answers were arrived at. If necessary, use a calculator to decide which answers are correct, but do not do that before having a full discussion of student strategies.

The key mathematical ideas here rest on the definition of exponentiation as repeated multiplication.

In answering #8-9, students may see the real power of scientific notation: it is possible to get reasonable answers by using mental arithmetic!

### 7. Multiplication, Division, and Scientific Notation

This is a good time to summarize what has been learned in the unit, and to surface any lingering misconceptions. Making the poster will help students prepare for an end-of-unit assessment.

### 8. The Solar System

This lesson is another application of scientific notation to astronomy.

#1-2 are a good opportunity to practice some of this material.

#3-4 should make for an interesting discussion about the planets.

#1-4 are much easier than #5-9, which involve massive calculations, and which you may classify as extra credit.

# 1. Approximating Large Numbers

Powers provide a way of writing numbers in an equivalent form, that is often more compact.

Just as multiplication is repeated addition, raising a number to a power is repeated multiplication.

For example, $12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 = 2985984$

This can be written as $12^6$.
$12^6$ shorter to write than $12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12$ or 2985984, and it is shorter to key into a scientific calculator or computer.

**Notation:** Calculators use [^], [$x^y$], or [$y^x$] for *exponentiation* (raising to a power). Computers usually use [^].

Calculators can calculate with exponents that are not positive whole numbers. For example, it is possible to get a value for a number like $3^{-2.4}$ using the key for powers on your calculator. (Try it.) In this lesson, you will consider only *positive whole numbers* for exponents.

1. **Exploration.** Consider the number 123,456. Use your calculator to approximate the number as closely as you can with a power of 2 (the base is 2, and you have to find the best possible exponent.) How close can you get?

   Repeat with a power of 3, a power of 9, and a power of 10.

For problems **2. – 4.** find the whole number powers of 2, 3, 9, and 10 that are immediately below and above the given number.

**Example:** 747,610 (the population of Virginia in 1790, and the largest state population at that time)

| | | | |
|---|---|---|---|
| 747,610 is between | $2^{19}$ | and | $2^{20}$ |
| 747,610 is between | $3^{12}$ | and | $3^{13}$ |
| 747,610 is between | $9^{6}$ | and | $9^{7}$ |
| 747,610 is between | $10^{5}$ | and | $10^{6}$ |

2. 3,893,635 (the population of the United States in 1790)

| | | | |
|---|---|---|---|
| 3,893,635 is between | $2^{\square}$ | and | $2^{\square}$ |
| 3,893,635 is between | $3^{\square}$ | and | $3^{\square}$ |
| 3,893,635 is between | $9^{\square}$ | and | $9^{\square}$ |
| 3,893,635 is between | $10^{\square}$ | and | $10^{\square}$ |

3. 50,456,002 (the number of people who voted for George W. Bush in 2000)

| | | | |
|---|---|---|---|
| 50,456,002 is between | $2^{\square}$ | and | $2^{\square}$ |
| 50,456,002 is between | $3^{\square}$ | and | $3^{\square}$ |
| 50,456,002 is between | $9^{\square}$ | and | $9^{\square}$ |
| 50,456,002 is between | $10^{\square}$ | and | $10^{\square}$ |

4. 50,999,897 (the number of people who voted for Vice-President Al Gore in 2000)

50,999,897 is between $2^{\square}$ and $2^{\square}$

50,999,897 is between $3^{\square}$ and $3^{\square}$

50,999,897 is between $9^{\square}$ and $9^{\square}$

50,999,897 is between $10^{\square}$ and $10^{\square}$

5. **Discussion**: If we tried to approximate the large numbers in #2-4 by using a power of a small number, would we get a pretty close estimate, or a pretty bad one?

6. Give a shortcut to multiply any number by the following numbers, without a calculator:
   a. 10

   b. $10^3$

7. Explain how to write $10^6$ without exponents. No calculator!

8. Explain how to write 100,000 as a power of 10. No calculator!

9. **Challenge.** Look back at #2-4. For one of the numbers 2, 3, 9, and 10, it is possible to find the answers without a calculator. Which number is it? Explain.

## 2. Closer Approximations

It is possible to combine powers with multiplication to get approximations that are closer than those you were able to get using only powers.

For example, the speed of light is approximately 186,282 miles per second. This number is more than $2^{17}$ and less than $2^{18}$, since:

$2^{17} = 131{,}072$ $2^{18} = 262{,}144$

By multiplying 131,072 by a number less than the base, which is 2, it is possible to get much closer to 186,282:

$1.2 \cdot 131{,}072 = 157{,}286.4$ (too small)
$1.5 \cdot 131{,}072 = 196{,}608$ (too large)
$1.4 \cdot 131{,}072 = 183{,}500.8$ (too small, but pretty close!)

Multiplying $2^{17}$ by 1.4 gives us a good approximation of the speed of light.

1. Using trial and error, find an approximation of the speed of light that is even closer than $1.4 \cdot 2^{17}$ by multiplying $2^{17}$ by a number that is between 1.4 and 1.5.

### Other Ways of Approximating

You can approximate the speed of light in many other ways using powers of 2. For example:

$93141 \cdot 2^1 = 186{,}282$
$46570 \cdot 2^2 = 186{,}280$
$23280 \cdot 2^3 = 186{,}240$
$45.5 \cdot 2^{12} = 186{,}368$

$2^{17}$ was used in the example above instead of some other power of 2 because it is the *largest* power of 2 that is less than 186,282. We approximated 186,282 by *multiplying the largest possible power of 2, by a number between 1 and 2*.

2. Write an approximation for the speed of light using a power of 3 multiplied by a number between 1 and 3. (Hint: Begin by finding the largest power of 3 that is less than 186,282.)

3. Write an approximation for the speed of light using a power of 9 multiplied by a number between 1 and 9.

4. Write an approximation for the speed of light using a power of 10 multiplied by a number between 1 and 10.

5. Combine a power and multiplication to get a close approximation for the length of the Earth's equator, which is 24,902 miles to the nearest mile. You can use any base for the power, but multiply the power by a number that is between 1 and the base.

# 3. Scientific Notation

1. Write 100 and 1000 as powers of 10.

| Power | Name |
|---|---|
| $10^6$ | million |
| $10^9$ | billion |
| $10^{12}$ | trillion |
| $10^{15}$ | quadrillion |
| $10^{18}$ | quintillion |
| $10^{21}$ | sextillion |
| $10^{100}$ | googol |

There are common names for some of the powers of ten. “Billion” in the U.S. means $10^9$, but in Britain it means $10^{12}$. The table above gives the common names used in the U.S. for some powers of ten.

2. Someone might think a billion is two millions, and a trillion is three millions. In fact, a billion is how many millions? A trillion is how many millions? Explain.

**Definition:** To write a number in *scientific notation* means to write it as a power of 10 multiplied by a number between 1 and 10. This is the most common way of writing large numbers in science and engineering.

3. Write each number in scientific notation.
    a. one million

    b. 67 million (the average distance from the sun to Venus in miles)

    c. 5.3 billion (an estimate of the world's population in 1990)

    d. twenty billion

    e. three hundred trillion

4. Why do you think 10 is used for the base rather than some other number? Use examples in your explanation.

5. **Research.** Find four large numbers that measure some real quantity. They should all be larger than 10,000,000. The Web, encyclopedias, almanacs, and science books are good sources of such numbers.
    a. Tell what each number measures.

    b. Write each number in scientific notation.

# 4. Scientific Notation on a Calculator

Calculators can only display numbers up to a certain number of digits. For some calculators, ten digits is the limit.

1. What is the limit for your calculator?

2. What is the smallest power of 2 that forces your calculator into scientific notation?

On many calculators, the answer to #2 is $2^{34}$, which, according to the calculator is equal to

1.717986918 $^{10}$ or 1.717986918E10 or $1.717... \times 10^{10}$

The expression on the left does *not* mean $1.717986918^{10}$, even though that's what it looks like. It is just calculator shorthand for $1.717986918 \cdot 10^{10}$. (The actual value is 17179869184, which is too long to fit, so the calculator gives the approximate value of 17179869180, expressed in scientific notation. For a number this large, this represents a very small error.

3. On a certain calculator, a power of 2 is displayed as 2.814749767E14 .
   a. Is the exponent of 2 for that power greater or less than 14?

   b. Is it greater or less than 34?

   c. Experiment on your calculator to find the exponent.

4. Find a power of 4 and a power of 8 that are also displayed as [2.814749767E14].

5. Find powers of 3, 9, 27, and 81 that are displayed in scientific notation, in the form ____ $\cdot 10^{17}$. Can you find more than one solution?

There are three ways to enter numbers in scientific notation into a calculator. For example, to enter $2 \cdot 10^3$, you can key in 2 [*] 10[^] 3, or 2 [*][$10^x$] 3, or (depending on the calculator) 2 [EE] 3, or 2[EXP] 3.

6. Try all the methods listed above that are available on your calculator. In each case, the calculator should respond with [2000] after you press [=] or [ENTER].

7. Explain the purpose of the [^] and [EE] (or [EXP]) keys. How are they different?

## 5. Look, Up in the Sky!

The table shows the ten brightest objects in the sky, and their *average* distance from Earth, in miles. (The objects are listed in order of average brightness as seen from Earth.)

| | **Distance** (miles) |
|---|---|
| Sun | $9.29\ (10^{7})$ |
| Moon | $2.39\ (10^{5})$ |
| Venus | $9.30\ (10^{7})$ |
| Jupiter | $4.84\ (10^{8})$ |
| Sirius | $5.11\ (10^{13})$ |
| Canopus | $5.76\ (10^{14})$ |
| Arcturus | $2.12\ (10^{14})$ |
| Mars | $1.42\ (10^{8})$ |
| Vega | $1.59\ (10^{14})$ |
| Saturn | $8.88\ (10^{8})$ |

1. If you were to divide the objects into two groups, based only on the value of the exponents of 10, what would be in each group? What is the actual significance of the two groups?

For each pair of objects given in #2-4, answer both questions comparing their distances from the Earth. (Since the Moon, planets, and stars are always in motion, these comparisons are not about actual distances at any one time, but the comparisons are still meaningful in a general way.) If your answer is greater than 10,000, give it in scientific notation:

a. On average, the second object is how many miles further from Earth than the first?
b. On average, the second object is how many times as far as the first?

2. The Moon, Venus.

3. The Sun, Sirius.

4. Sirius, Canopus.

## 6. Without a Calculator

1. Convert these numbers to ordinary decimal notation and add them without a calculator.
   a. $(4 \cdot 10^7) + (5 \cdot 10^6)$

   b. $(40 \cdot 10^6) + (5 \cdot 10^6)$

2. Compare the two computations in the previous problem. Which would have been easy to do without converting to ordinary decimal notation? Explain.

Without a calculator, it is not easy to add and subtract in scientific notation. One way is to revert to ordinary decimal notation. Another is to write the two quantities with a common exponent for 10 as was done in Problem 1b.

3. Add or subtract:
   a. $6.2 \cdot 10^3 + 5 \cdot 10^6$

   b. $6.2 \cdot 10^6 - 5 \cdot 10^3$

   c. $6.2 \cdot 10^5 + 5 \cdot 10^3$

   d. $6.2 \cdot 10^3 - 5 \cdot 10^6$

Without a calculator, it can be tedious to multiply and divide large numbers. However, if the numbers are written in scientific notation it is possible to estimate the size of the answer.

For the following problems:

a. Multiply or divide.
b. Write your answer in scientific notation.

4. $(3 \cdot 10^5) \cdot (6 \cdot 10^3)$

5. $(3 \cdot 10^3) \cdot (6 \cdot 10^5)$

6. $(6 \cdot 10^6) \div (3 \cdot 10^3)$

7. $(3 \cdot 10^6) \div (6 \cdot 10^3)$

For #8-9, use the numbers from Lesson 5 to answer the question "On average, the second object is how many times as far as the first?"

a. write down an estimate of the answer without a calculator
b. see how close you were with the help of the calculator

8. The Moon, Saturn.

9. The Sun, Canopus.

## 7. Multiplication, Division, and Scientific Notation

Work with a partner to prepare a poster about scientific notation.

Include a definition of scientific notation, with examples.

Give examples of numbers that appear to be written in scientific notation, but are not.

Explain how to multiply and divide numbers in scientific notation. Include an explanation of this statement:

Multiplying two numbers written in scientific notation involves a multiplication and an addition.

You should also include and explain a similar (but different) statement about dividing.

As you prepare your poster, discuss this with your partner: does your method work for

multiplying $8\ (2^6)$ by $4\ (2^4)$?

dividing $8\ (2^6)$ by $4\ (2^4)$?

These numbers are not in scientific notation, but the math should still work!

Include an explanation of this and other examples in your poster.

You may use color and arrows to make your poster as clear as possible to a student who is learning about scientific notation.

# 8. The Solar System

The table below gives the diameter and average distance from the Sun in kilometers (km) of each of the planets in the solar system. The Sun's diameter is also shown.

| | Diameter | Average Distance from Sun | Moons |
|---|---|---|---|
| Sun | $1.39\ (10^6)$ | | |
| Mercury | $4.88\ (10^3)$ | 57,700,000 | 0 |
| Venus | $1.21\ (10^4)$ | 108,150,000 | 0 |
| Earth | $1.23\ (10^4)$ | 150,000,000 | 1 |
| Mars | $6.79\ (10^3)$ | 227,700,000 | 2 |
| Jupiter | $1.43\ (10^5)$ | 778,300,000 | 17 |
| Saturn | $1.20\ (10^5)$ | 1,427,000,000 | 22 |
| Uranus | $5.18\ (10^4)$ | 2,870,000,000 | 15 |
| Neptune | $4.95\ (10^4)$ | 4,497,000,000 | 3 |
| Pluto | $6.00\ (10^3)$ | 5,900,000,000 | 1 |

1. Convert the diameters to normal decimal notation.

2. Convert the distances to scientific notation.

3. Divide the planets into groups according to:
   a. Their diameters. How many groups are there? Explain.

   b. Their average distance from the sun. How many groups are there? Explain.

   c. Their number of moons. How many groups are there? Explain.

4. Compare the groups you created in the previous problem. Find a way to combine your decisions into an overall division of the planets into two or three groups, by "type of planet." Name each group, and list its characteristics in terms of the data in the table.

5. **Challenge:** Light travels approximately 299,793 kilometers per second. Show your calculations, and give your answers in scientific notation. How far does light travel in:
   a. one minute

   b. one hour

   c. one day

   d. one year

6. Abe remembers learning in elementary school that it takes about 8 minutes for light to travel from the sun to the earth. Figure out whether he remembers correctly. Show your calculations.

7. Which planets, if any, does it take light from the sun more than one day to reach?

8. When Pluto is at its mean distance from the sun, how long does it take light from the sun to reach it?

9. An Astronomical Unit is the distance from the Earth to the Sun. What is Pluto's distance from the Sun in Astronomical Units?