Building Block Numbers

Food for Thought

1. Sally tried to order 13 chicken "nuggets" at the fast food store. The employee informed her that she could only order 6, 9, or 20. Sally realized she had to decide between ordering 6 + 6 = 12, or 6 + 9 = 15.
   - ◊ What numbers of nuggets can be ordered by combining 6, 9, and 20?
   - ◊ What numbers cannot be ordered?
   - ◊ What is the greatest number that cannot be ordered?

Two Building Blocks

In the previous problem, 6, 9, and 20 were our building block numbers. It is interesting that all numbers beyond a certain number can be ordered. To think about this further, let us look at some more examples, using only two building block numbers.

2. You have an unlimited supply of dimes (10 cents) and quarters (25 cents). What amounts can be obtained, and what amounts cannot be obtained by combining them?

3. At Fred's Kitchen Supply, cabinets are available in two lengths: 3 feet and 5 feet. By putting cabinets end to end, walls of different lengths can be accommodated. Imagining that kitchens can be arbitrarily large, what length walls are possible to line exactly with cabinets? What lengths are impossible?

4. What numbers can be obtained by adding the numbers 6 and 9 as many times as you want? What numbers cannot be obtained?

5. In 1958, it cost 4 cents to mail a letter in the United States. In 1963, it cost 5 cents. Imagine you have an unlimited supply of 4 and 5-cent stamps. What amounts can you make? What is the largest amount you cannot make?

6. Generalize: By now, you are probably aware that some pairs of building block numbers work better than others.
   a. Which types of pairs allow us to build every number beyond a certain point?
   b. For the other pairs, what do they allow us to build?

A Visual Strategy

It would be interesting to know how to find the greatest impossible number for a given pair of building block numbers, if it exists. We will investigate the problem using 5 and 7.

7. Write the numbers from 0 to 39 in an array like this:

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 0  1  2  3  4
 5  6  7  8  9
10 11 12 13 14
.. .. .. .. ..
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8. Then, circle all the numbers we can get by adding 5's and 7's. Follow the following process:
   - ◊ Circle the multiples of five, since we can get them. (Remember that 0 is a multiple of 5.)
   - ◊ Circle the numbers that are equal to 7 plus a multiple of 5.
   - ◊ Circle the numbers that are equal to 14 plus a multiple of 5.
   - ◊ Continue in this fashion, using 21, 28, etc as your starting point, until you have circled enough numbers to know what numbers can and cannot be built from 5 and 7.

Think about the strategy we followed: first, we organized the numbers in five columns (why five?), and then we circled the numbers we can get in an organized fashion.

9. Apply this method if the building blocks were
   a. 5 and 6?
   b. 4 and 6?

10. What went wrong in the case of 4 and 6?

11. Follow the same procedure in the case of generic building blocks a and b, and try to find a formula for the greatest impossible number in terms of a and b.

Adapted from *Algebra: Themes, Tools, Concepts*, © Anita Wah and Henri Picciotto (1994)