Constant Sums

You will need: graph paper

At the Gas Station

When Oliver and Alice pulled up to the self-serve island at Jacob's gas station, they noticed a new sign:

`Buy gas card in office`

They went into the office, which was decorated with great photographs and cartoons. The attendant, Harold, explained to them that he could sell them a gas card for any amount from 5 to 100 dollars. They would put it into the special slot in the pump, and pump gas as usual. The value of the card would automatically go down, and a display on the pump would indicate how much value was left in the card. After getting gas, there would be no need to go back to the office, unless they wanted to trade the card back for cash. (This could only be done if the card had less than $5 left on it.) Or, they could use the remaining money left on the card the next time they stopped at Jacob's.

Jacob's Gas

Price Per Gallon	Amount Spent
$ 1.099	$ 3.41
Gallons Pumped	**Value Left on Card**
3.1	$ 11.59

1. Oliver and Alice started pumping gas. Look at the pump display shown in the figure above. How much did they pay for their gas card?

2. List at least five other pairs of numbers that will appear on the two dials on the right while they are pumping gas.

3. When exactly 11 gallons have been pumped, what numbers will appear on the four dials?

4. **Generalization.** When D dollars have been spent, what is the value left on the card?

5. When G gallons have been pumped,
   a. how many dollars have been spent?
   b. what is the value left on the card?

Graphs of Constant Sums

6. a. On a pair of axes, plot these (x,y) pairs: (2,4) (4,2) (-1,7) (8,-2)
   b. In words, we could describe the pattern of the (x,y) pairs by saying that the sum of x and y is always 6. How would you write this using algebra?
   c. Write three more (x,y) that fit this pattern and add the points to your graph.
   d. Connect all the points with a line or curve.

7. a. Find points such that $x + y < 6$. Where are they in relation to the graph above?
   b. Repeat for $x + y = 6$
   c. Repeat for $x + y > 6$

8. Find a point (x,y) such that $x = y$ and $x + y = 6$. Label it on the graph.

9. Choose a positive value for S and make a table of (x,y) pairs that satisfy the equation $x + y = S$. Use your table to make a graph.

10. € Experiment with some other constant sum graphs. Try several different positive values for S. For each one, make a table of at least five (x,y) pairs having the sum S. Then draw a graph. Draw all your graphs on the same pair of axes.

11. € Do any of the lines go through the origin? If not, do you think you could pick a number for your sum so that the line would go through the origin? Explain.

12. € Repeat your investigations for equations of the form x + y = S where S is negative. Keep a record of what you try, using tables and graphs.

13. **Discussion.** These questions are about constant sum graphs.
    - Were the graphs straight lines or curved, or were there some of each?
    - Without drawing the graph, could you now predict which quadrants the graph would be in if you knew the value of S? Explain.
    - Without drawing the graph, could you predict the x-intercepts and y-intercepts of the graph if you knew the value of S? Explain.
    - What determines whether the graph slopes up or down as it goes from left to right? Could you predict without graphing if you knew the value of S? Explain.
    - Do any of your graphs intersect each other? If so, which ones? If not, why not?

Constant Products

You will need: graph paper

Miles Per Gallon

If you plan to take a trip of 100 miles, the amount of gas you need depends on how many miles per gallon your vehicle gets. Some very large recreational vehicles get only about 5 miles per gallon, while a scooter can get 100 miles per gallon.

1. Copy and complete the table to show how many gallons of gasoline you should buy if your vehicle gets the mileage indicated. Continue the table up to 100 miles per gallon.

mileage (miles per gallon)	gasoline needed (gallons)	total trip distance (miles)
5		100
10.5		100
20		100

2. Graph the (x,y) pairs in the first two columns of the table.

3. Describe your graph in words. If you were to extend your graph, would it go through the origin? Would it touch or cross the axes? Explain.

Connecting the dots

4. Make a table containing these points and plot the (x,y) pairs on a Cartesian graph.
(2,12) (3,8) (4,6) (8,3)

5. Describe the pattern of the (x,y) pairs above:
a. in words b. using algebra

6. a. Write five more (x,y) pairs that fit this pattern and add the points to your table and graph. Use positive values for x. Include some fractional values.
b. Add five more (x,y) pairs to your table and graph. This time use negative values for x, including some fractional values.

7. Study the points on your graph. If necessary, add more points so that you can answer the following questions:
a. Which quadrants do your points lie in? Why?
b. Can you find a point on the y-axis that fits the pattern? Can you find a point on the x-axis? Explain.
c. If you were to connect the points with a smooth curve, would the curve go through the origin? Explain.

8. Add to your graph a point that fits the pattern and:
a. has an x-value less than 1/2
b. has a y-value less than 1/2
c. has an x-value greater than 24
d. has a y-value less than -24

9. Study your answers to the previous problems. Then very carefully connect the points with a curve. Your curve should have two parts *that are not connected* to one another.
a. Describe the graph.
b. Explain why the two parts are not connected.

10. For each part, find several pairs of numbers (x,y) that satisfy the description. Plot these points on your graph.
a. x is positive and xy is more than 24
b. x is positive and xy is less than 24
c. x is negative and xy is more than 24
d. x is negative and xy is less than 24

11. * Plot a point (x,y) such that xy = 24 and x = y.

We could call the curve you drew in Problem 9 a "constant product" graph, since the product of the coordinates of every point is the same number. We could graph many other "constant product" graphs of the form xy = P, where P could be any number we choose.

12. € Experiment with the graphs of some equations of the form xy = P. Try several different positive values for P. Then try several different negative values for P. For each one, make a table of at least eight (x,y) pairs having the product. Then draw a graph. Draw all your graphs on the same pair of axes.

13. **Discussion.** These questions are about constant product graphs.
    - What is the shape of the graph?
    - Without drawing the graph, could you now predict which quadrants the graph would be in if you knew the value of P? Explain.
    - Do any of the graphs go through the origin? If not, do you think you could find a value of P so that the graph would go through the origin? Explain.
    - Where can you find points whose product is not P?
    - Comment on anything you notice about the x-intercepts and y-intercepts.
    - Do any of your graphs intersect? Explain why or why not.

Analyzing Graphs

You will need: graph paper

Constant Products

1. a. On the same pair of axes, graph the constant product function $xy = 24$ and the constant sum function $x + y = 10$.
   b. Find and label the points where these two graphs intersect.
   c. Add the graph of $x + y = 4$ to the same pair of axes. Does it intersect either graph?

2. If possible, factor each trinomial.
   a. $x^2 + 10x + 24$
   b. $x^2 + 4x + 24$

3. € Explain the relationship between Problem 1 and Problem 2.

4. Make a large graph of the constant product $xy = 36$. Show both branches on your graph.

5. On the graph of $xy = 36$, find two (x,y) pairs whose sum is 13. Plot and label these points, and connect them with a straight line. What is the equation of the line connecting these two points?

6. Add to your graph several lines of the form $x + y = S$, where S is an integer, as described below. Draw at least three:
   a. that intersect the graph of $xy = 36$ in the first quadrant. (Label the graphs and the points of intersection.)
   b. that intersect the graph of $xy = 36$ in the third quadrant. (Label the graphs and the points of intersection.)
   c. that never intersect the graph of $xy = 36$.

7. € Consider the expression x^2 + ___x + 36. What numbers could you put in the blank to get a trinomial that can be factored? Explain your answer, giving examples.

Constant Sums

8. Make a large graph of the constant sum $x + y = 12$.

9. a. Find many (x,y) pairs whose product is 20.
   b. Plot these points, and connect them with a smooth curve.
   c. What is the equation of the curve?
   d. Where does it meet the graph of $x + y = 12$?

10. Add to your graph several curves with equations of the form $x \cdot y = P$, where P is an integer, as described below. Draw at least three:
    a. that intersect the graph of $x + y = 12$ in the first quadrant.
    b. that intersect the graph of $x + y = 12$ in the second and fourth quadrants.
    c. that never intersect the graph of $x + y = 12$.

11. € Consider the expression $x^2 + 12x$ + ___. What numbers could you put in the blank to get a trinomial that can be factored? Explain your answer, giving examples.

12. **Report.** Summarize what you discovered in this lesson. Concentrate on the question: *How are the points of intersection of constant sum and constant product graphs related to factoring trinomials?* Use examples and illustrate your report with graphs. (The examples given in this lesson involved only positive whole numbers for the sums and products. In your report, you may use negative numbers or zero.)