

Perimeter of Polyominoes

You will need:

graph paper



SHORTEST AND LONGEST PERIMETER

For polyominoes with a given area, there may be more than one perimeter. In this section, you will try to find the shortest and the longest perimeter for each given area.

- Copy this table, extend it to area 24, and fill it out. (A few rows have been done for you.) Experiment on graph paper as much as you need to, and look for patterns.

| Area | Perimeter | |
|------|-----------|---------|
| | Shortest | Longest |
| 1 | 4 | 4 |
| 2 | 6 | 6 |
| 3 | | |
| 4 | 8 | 10 |
| 5 | | |
| ... | | |

- What patterns do you notice in the table? Explain.
- Describe the pattern for the perimeter of a polyomino of area A , having:
 - the longest perimeter;
 - the shortest perimeter.
- For a polyomino having a given area, what perimeters are possible between the shortest and longest? (For example, for

area 4, the minimum perimeter is 8, and the maximum is 10. Is it possible to have a perimeter of 9?)

- What perimeters are possible for area 9?

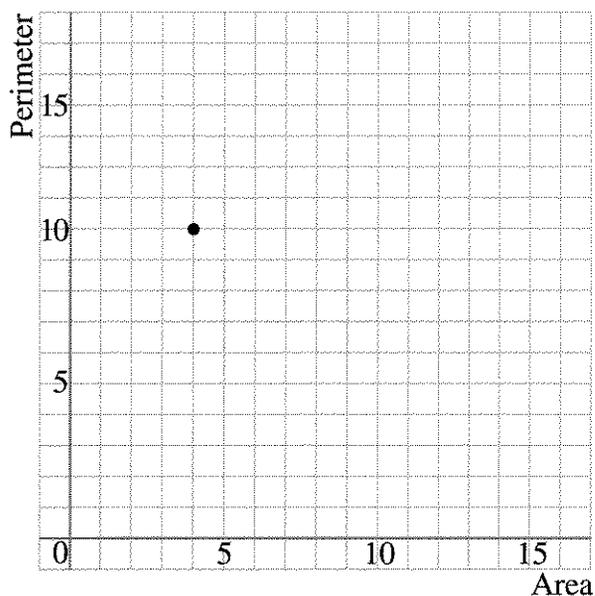
MAKING PREDICTIONS

Mathematics is the science of patterns. Discovering a pattern can help you make predictions.

- Predict the longest possible perimeters for polyominoes having these areas. If the number is not too big, experiment on graph paper to test your predictions.
 - 36
 - 40
 - 100
 - 99
 - 101
 - 1000
- Explain your method for answering problem 6.
- Predict the shortest possible perimeters for polyominoes having these areas. If the number is not too big, experiment on graph paper to test your predictions.
 - 36
 - 40
 - 100
 - 99
 - 101
 - 1000
- Explain your method for answering problem 8.

MAKING A GRAPH

- On graph paper, draw a horizontal axis and a vertical axis. Label the horizontal axis *Area* and the vertical axis *Perimeter*, as in the following graph. Extend them as far as you can, to at least 25 units for area and 55 units for perimeter.



Definition: The point where the axes meet is called the *origin*.

For the following problems, you will need the numbers you found in the table in problem 1.

11. For each area, there is one number for the longest perimeter. For example, the longest perimeter for an area of 4 is 10. This gives us the number pair (4, 10). Put a dot on the graph at the corresponding point. (Count 4 spaces to the right of the origin, and 10 spaces up.) Do this for all the *area and longest perimeter* points on the table.
12. Describe what the graph looks like.

13. Using the same axes, repeat problem 1 with the numbers for area and shortest perimeter. One dot would be at (4, 8).

14. Describe what the graph looks like.

INTERPRETING THE GRAPH

15. Explain why the first set of points is higher on the graph than the second set.
16. As the area grows, which grows faster, the longest perimeter or the shortest perimeter? What happens to the gap between the two?
17. Use the graph to figure out how many different perimeters are possible for an area of 25. Explain how you did it.
18. Use the table you made in problem 1 to answer problem 17. Explain how you did it.
19. Use the graph to check whether there is a polyomino having area 15 units and perimeter 20. Explain how you did it.
20. Use the table you made in problem 1 to answer problem 19. Explain how you did it.

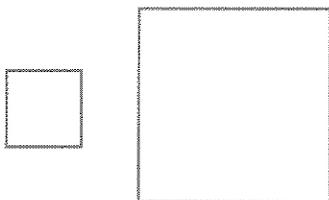
In this lesson you used patterns, tables, and graphs to help you think about a problem. This is an important skill which you will develop throughout this course.

PREVIEW UNITS AND DIMENSIONS

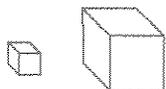
Length is measured in linear units, such as the inch (in.) or centimeter (cm). Length refers to one dimension.



Area is measured in square units, such as the square inch (in.², or sq in.) or square centimeter (cm²). Area refers to two dimensions.



Volume is measured in cubic units, such as the cubic inch (in.³, or cu in.) or cubic centimeter (cm³, or cc). Volume refers to three dimensions.



21. Divide the following units into three groups according to what they measure: length, area, or volume.

- | | |
|--------------|----------------|
| a. acre | b. fluid ounce |
| c. foot | d. gallon |
| e. kilometer | f. liter |
| g. meter | h. mile |
| i. pint | j. quart |
| k. yard | |

22. For each unit listed in problem 21, name something that might be measured with it. For example, for (a), the area of a farm could be measured in acres.