

Minus and Opposites

You will need:

the Lab Gear



THREE MEANINGS OF MINUS

The *minus* sign can mean three different things, depending on the context.

- It can mean **negative**. In front of a positive number, and only there, it means negative. Example: -2 can mean negative 2.
 - It can mean **opposite**. The opposite of a number is what you add to it to get zero. Example: -2 can mean the opposite of 2, which is negative 2, since $2 + -2 = 0$. Likewise, $-x$ means the opposite of x , and $x + -x = 0$.
 - It can mean **subtract**. Between two expressions, it means subtract the second expression from the first one. For example, $x - 3$ means subtract 3 from x .
1. For each of the following, write an explanation of what the minus sign means.
 - a. $y - 5$
 - b. $-(5x + 1)$
 - c. -2
 - d. $-x$
 2. Write the value of $-x$ if:
 - a. $x = 2$;
 - b. $x = -3$.
 3. True or False? (Explain your answers.)
 - a. $-x$ is always negative.
 - b. $-x$ can be positive.

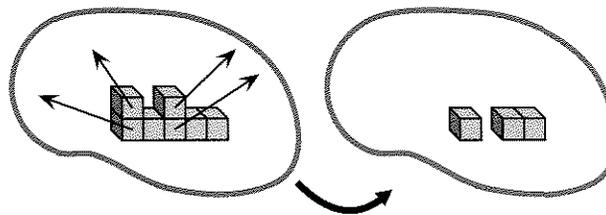
Notation: In this book, the minus sign meaning *negative* or *opposite* will be smaller than the one for subtract. In handwriting, this is not necessary. However some calculators use different keys for the two meanings: $\boxed{-}$ for subtraction, and $\boxed{(-)}$ or $\boxed{+/-}$ for *negative* or *opposite*.

There are two ways of showing minus with the Lab Gear: upstairs and the minus area.

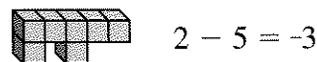
UPSTAIRS

Rule: Any blocks placed on top of other blocks are preceded by a minus sign.

This figure shows $5 - 2$. Notice that the *uncovered* part of the bottom block equals 3. If you remove matching upstairs and downstairs blocks, you will be left with three downstairs blocks. This is how we show $5 - 2 = 3$ with upstairs and downstairs blocks.

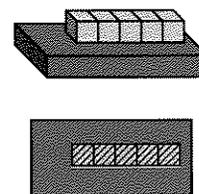


This figure shows $2 - 5$. If you mentally remove matching blocks downstairs and upstairs, you are left with 3 upstairs blocks, or -3 . We can only do this mentally, however, since blocks cannot float in mid-air.



Do not stack Lab Gear blocks more than two levels high. Two levels are enough to illustrate many ideas of algebra and will keep things clear. More would be confusing.

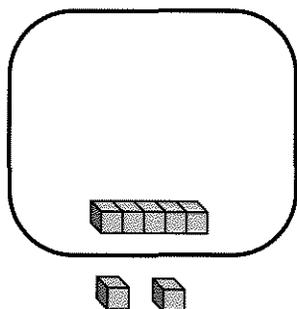
Subtraction with variables is shown in the same way. The amount being subtracted must be placed upstairs. Note that upstairs blocks are shaded in the 2-D sketch.



The upstairs method of showing minus is important and useful, but it is limited; it cannot easily be used to show minus when it means *negative* or *opposite*.

THE MINUS AREA

Look at your workmat. The rectangles with rounded corners represent the **minus areas**. The whole collection of blocks inside the minus area is preceded by a minus sign. For example, $2 - 5$ can be shown this way. (Here the minus sign means *subtract*.)



If you remove the matching blocks inside and outside the minus area, you will be left with three blocks inside the minus area, or -3 . (Here the minus sign means *negative*.)

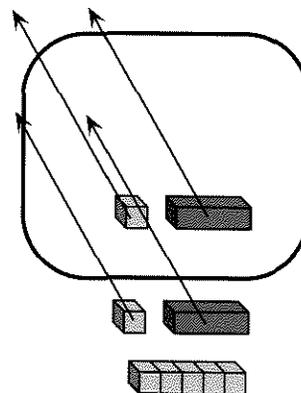
4. Sketch how you would show each quantity on the workmat. You may need to use upstairs in some of the problems.
- a. $5 - x$ b. $x - 5$
 - c. $-(x + 5)$ d. $-(5 - x)$
 - e. -5

5. Summary

- a. Explain, using examples, how the minus area can show all three meanings of minus.
- b. Which of the three meanings does the upstairs method show best? Explain.
- c. Put some blocks in the minus area, including some blocks upstairs. Sketch. What quantity does this arrangement represent?

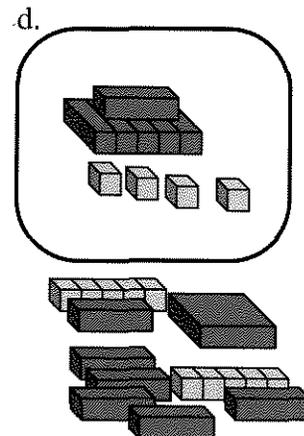
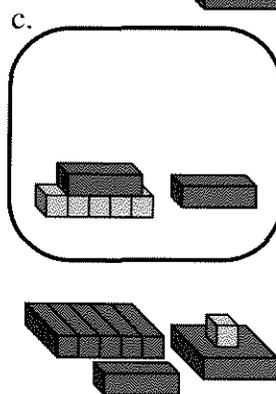
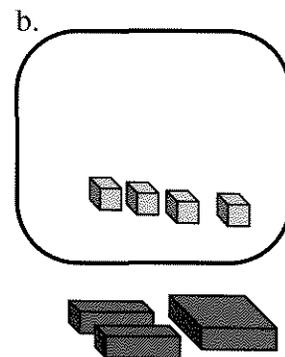
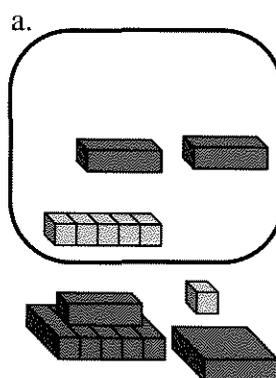
REMOVING OPPOSITES

When the quantities inside and outside the minus area are the same, they add up to zero and can be removed. For example, the figure shows that $5 + x + 1 - (x + 1) = 5$.



Similarly, matching upstairs and downstairs quantities add up to zero, and can be removed.

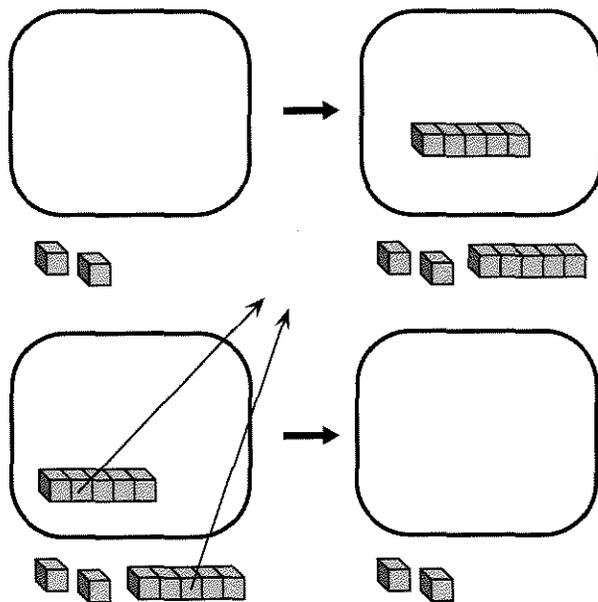
6. Two of these four figures represent the same quantity. Which two? Explain.



ADDING ZERO

The number 2 can be shown most simply with two 1-blocks outside the minus area. However, sometimes it is useful to show the number 2 using more blocks.

For example, after adding a five-block in the minus area and a five-block outside, the figure still shows 2. Since 5 and -5 are opposites, their sum is zero, so we really added zero. The technique of adding zero is useful in many situations.



7. Sketch two other ways to show the number 2.

8. Sketch or explain how to show -9 with:
 - a. three blocks;
 - b. five blocks;
 - c. seven blocks.
9. Sketch or explain how you would show 5 with:
 - a. 3 blocks;
 - b. 11 blocks.
10. Can you show 5 with any number of blocks? Can you show it with 100 blocks? With 101 blocks? Explain your answers.
11. a. Show $x - 1$ in at least three different ways. Sketch or explain.
 b. Show $1 - x$ in at least three different ways. Sketch or explain.

MINUS PUZZLES

12. Nineteen numbers can be shown with exactly two yellow blocks. What are they?
13. Find three ways to show -4 using only a 5-block and a 1-block. Sketch or explain.
14. Find four ways to show 3 with three blocks. Sketch or explain.
15. Find four ways to show -8 with four blocks. Sketch or explain.
16. Make up a puzzle like the above for a classmate. Solve a classmate's puzzle.