Three Dimensions

Just as we used the area of a rectangle to help us model multiplication of two factors, we can use the volume of a box to help us model multiplication of three factors.

For example, \(5 \times x \times y\) can be shown like this.

But another way to show it could be:

1. Use the Lab Gear to show how \(x^2y\) can be seen as a product of:
   a. three factors;
   b. two factors;
   c. two factors in another way.

Associative and Commutative Laws

In a multiplication the factors can be grouped in any way. For example, \((-2 \cdot 3) \cdot 4 = -2 \cdot (3 \cdot 4)\). This is called the associative law for multiplication.

In a multiplication the factors can be multiplied in any order. For example, \(5 \cdot (-6) = (-6) \cdot 5\). This is called the commutative law for multiplication.

2. Using six \(xy\)-blocks, it is possible to make a rectangle in four different ways. Find all four rectangles, and write a multiplication equation for each.

3. Using six \(xy\)-blocks, it is also possible to make a three-dimensional box. There are many such boxes. Find five, and write at least two multiplications for each one.

4. Summary Explain how problems 2-3 about \(6xy\) provide examples of the associative and commutative laws for multiplication.

How Many Terms?

5. Exploration After combining like terms, how many terms does the product have for each of the following multiplications? Is there a pattern? You may use the Lab Gear.
   a. \(2x \cdot 3x\)
   b. \(2(x + 3)\)
   c. \(2x(x + 3x)\)
   d. \((3 + x)(x + 2)\)
The figure shows \((x + 3)(x + 5)\).

The resulting rectangle is made up of four smaller rectangles. The area of each one is shown in the figure.

6. a. Which two rectangles are made up of the same kind of block? 
b. What is the answer to the multiplication \((x + 3)(x + 5)\)? Combine like terms in your answer. How many terms are in your final answer?

7. a. Use the corner piece to model the multiplication \(3x(x + 5)\). Sketch it, showing the resulting rectangle. 
b. On your sketch, write the area of each of the smaller rectangles that make up the larger rectangle. 
c. Write the result of the multiplication \(3x(x + 5)\). Combine like terms. 
d. How many terms are in your final answer?

8. Repeat problem 7 for \((x + 3)(x + y + 5)\).

9. Repeat problem 7 for \((x + y + 3)(x + y + 5)\).

10. Use the Lab Gear to model a multiplication problem that has four terms in the final answer. Sketch the blocks and write the multiplication.

**MAKE A RECTANGLE**

Take blocks for each expression.

a. Arrange them into a rectangle. 
b. Write a multiplication equation of the form \(\text{length times width equals area}\).

11. \(xy + 5y\) 
12. \(xy + 7x\) 
13. \(7y + 7x\) 
14. \(x^2 + 7x\) 
15. \(x^2 + 7x + xy\) 
16. Do not use the Lab Gear for this problem. Write the addition \(y^2 + 2xy + 3y\) as a multiplication. Explain how you solved the problem.

In problems 17 and 18, take blocks for each expression.

a. Arrange them into a rectangle. 
b. Write a multiplication equation of the form \(\text{length times width equals area}\).

17. \(x^2 + 7x + 6\) 
18. \(x^2 + 7x + 10\)