Introduction to Inequalities

You can tell which of two numbers is greater by their positions on the number line.

The number that is greater is farther to the right. The number that is less is farther to the left.

Notation: The symbol for less than is <. For example, -5 < 3, 0 < 7, and -6 < -2. The symbol for greater than is >. For example, 6 > 3, 0 > -2, and -5 > -9.

1. Use the correct symbol.
   a. -5 ? -7
   b. -5 ? -1

This workmat shows two expressions.
\[ x + 4 - 5 - (x + 5) \] and \[ 10 + 2x - 1 - (2x - 1) \]

Which is greater? The question mark shows that this is unknown.

2. Put out blocks to match the figure. Simplify both sides. Write an expression for the blocks that remain on the left side. Write an expression for the blocks on the right side. Which side is greater? Show your answer by writing the correct inequality sign between the two expressions.

For each problem, put out blocks to match the figure, and
   a. write the two expressions;
   b. simplify both sides on the workmat;
   c. decide which side is greater or whether they are equal, and write the correct sign between the expressions.

3. [Diagram]

4. [Diagram]
5. To compare $2x - x + 5 - (5 - x)$ with $5 + 3x - 1 - (x - 3)$, first show the two expressions with the Lab Gear.

Your workmat should look like this.

6. Both sides include $2x$, but the right side is greater, as it also includes 7 more units. So we can write $2x < 2x + 7$.

Now compare these expressions.

7. Simplify both sides, then arrange the blocks in a logical manner to determine which side is greater.

8. Write both expressions as they are shown in this figure.

9. Simplify both sides, then arrange the blocks in a logical manner to determine which side is greater.
Your workmat should look like this.

In this case, it is impossible to tell which side is greater, because we do not know whether \( x \) is greater or less than 2.

For problems 10-13, write both expressions as they are given. Then simplify, using your blocks, and write the expressions in simplified form. Decide which side is greater, whether they are equal, or whether it is impossible to tell. Write the correct symbol or ?.

10.  

11.  

12.  

13.  

14. a. Substitute -1 for \( x \) in both expressions and tell which is greater.  
b. Substitute 3 for \( x \) in both expressions and tell which is greater.  
c. Find another value for \( x \) which makes \( 2x - 5 \) greater.  
d. Find another value for \( x \) which makes \( -3x + 6 \) greater.

15. For each of the following pairs of expressions, find two values of \( x \), one that makes the first expression greater and one that makes the second expression greater. Show all your calculations.  
a. \( 7x - 4 \)  \( 3x - 2 \)  
b. \( -2x + 6 \)  \( 8x - 4 \)  
c. \( x \)  \( -x \)
For each pair of expressions, write
A if the expression in column A is greater;
B if the expression in column B is greater;
? if you would have to know the value of
\( x \) in order to know which is greater.

Remember that \( x \) can have negative and fractional values. It may help to think about the
Lab Gear. In each case explain your answer, giving test values of \( x \) if it helps your
explanation.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>16. 7x</td>
<td>7x - 1</td>
</tr>
<tr>
<td>17. 7x + 1</td>
<td>-7x + 1</td>
</tr>
<tr>
<td>18. 7x + 1</td>
<td>7x - 1</td>
</tr>
</tbody>
</table>

22. Compare your answers to problems 16-20
with other students’ answers. Discuss your disagreements. If you disagree with
another student, try to find an example to show which answer is not correct.

23. Write an expression containing \( x \), that is
less than 4 when \( x \) is less than 9.

24. Write an expression containing \( x \), that is
less than 4 when \( x \) is more than 9.

25. Write an expression containing \( x \), that
less than 4 for all values of \( x \).

26. More codes
If the coding function is of the form \( y = mx \), it
is more difficult to encode and decode. (For
the letter values, see Thinking/Writing 3.A.)

a. Encode the word extra using \( y = 3x \).
b. What makes \( y = 2x \) a difficult code
to crack?

27. Decode the following sentence which was
encoded with \( y = 3x \). It may help to make
a table showing the matching of the plain-
text and coded alphabet.

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28. Encode the word multiplication with:
a. \( y = x \);
b. \( y = 2x \);
c. \( y = 13x \);
d. \( y = 26x \).

29. Decode the following message, which
was encoded with \( y = 2x \). It may help
to make a table showing the matching

30. What is the sum of the word algebra?

31. Find as many words as possible having
sum 100.