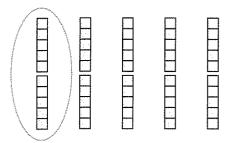


A MODEL FOR MULTIPLICATION

You cannot easily show multiplication by fractions with the Lab Gear, but the Lab Gear can help you think about it. For example, $(1/5) \cdot 50$ is read *one-fifth of fifty*. This means that we divide 50 into five parts and take one of them.

The diagram shows that $(1/5) \cdot 50 = 10$.



(2/5) is two of five parts, so $2/5 \cdot 50 = 20$.

- 1. Find a number you could multiply by 8 to get a number less than 8.
- 2. Without finding its value, decide whether *x* would be more or less than 1. Explain how you know.

a.
$$8 \cdot x = 50$$

b. $8 \cdot x = 5$
c. $8 \cdot x = 0.05$

3. Find the value of *x* for each equation in problem 2. (Hint: Remember that for any multiplication, there are two related divisions. You may use a calculator.)

A MULTIPLICATION SHORTCUT

4. Take 8, 3, and 2. They are three numbers whose product is 48. Another multiplication possibility is 6 • 4 • 2. Find as many ways of writing 48 as a product of three different numbers as you can. Do not use 1 as a factor.

- 5. Exploration Do not use 1 as a factor.
 - a. Write 2 as a product of two different numbers.
 - b. Write 4 as a product of four different numbers.
 - c. Write 6 as a product of six different numbers.
 - d. Write 12 as a product of twelve differ ent numbers.

Definition: The product of a number and i *reciprocal* is 1. Another way of saying this is, *the reciprocal of a number is the result dividing 1 by the number.*

Examples: $3 \cdot 1/3 = 1$ $2/3 \cdot 3/2 = 1$ $0.31 \cdot 100/31 = 1$

6. Explain how the reciprocals of 3, 2/ and 0.31 may have been found for the examples above. (No calculator was used

Guess the value of *x*, *without using your calc lator*. If you think about reciprocals you will have to do very little arithmetic.

7. a.
$$5 \cdot \frac{1}{5} \cdot x = 6$$

b. $4 \cdot x \cdot 9 \cdot \frac{1}{4} = 45$
c. $x \cdot 8 \cdot 7 = 8$
d. $x \cdot 8 \cdot 3 = 3$
e. $\frac{2}{3} \cdot x \cdot 3 \cdot \frac{1}{2} = 15$

8. a.
$$2 \cdot x \cdot 3 = 2$$

b. $x \cdot 2 \cdot 2 \cdot 9 \cdot 3 = 6$
c. $\frac{1}{5} \cdot (5x) \cdot 3 = 1$
d. $\frac{1}{5} \cdot (5x) = \frac{3}{5}$

9. Make up two more equations like problems 7 and 8 and solve them.





10. \bigcirc Find two numbers *a* and *b* that will satisfy each equation. *Don't use your calculator*. Instead, think about reciprocals. Do not use 1 for *a* or *b*.

a.
$$a \cdot b \cdot 14 = 28$$
 b. $a \cdot b \cdot 28 = 14$
c. $\frac{2}{3} \cdot a \cdot b = 10$ d. $a \cdot b \cdot 10 = \frac{2}{3}$

RECIPROCALS ON THE CALCULATOR

Most scientific calculators have a key for reciprocals: 1/x, or x^{-1} . (On calculators that do not have such a key, you can divide 1 by a number to find the number's reciprocal.)

- **11.** Find the reciprocal of:
 - a. 1/23; b. 0.456; c. 7.89.
- **12.** Report What is the result when you
 - a. multiply a number by its reciprocal?
 - b. divide a number by its reciprocal?

Be sure your results work for all numbers. Explain how you reached your conclusions.

- **13.** 1/82 < 0.0123 < 1/81. Explain.
- **14.** Find two consecutive whole numbers such that 0.00123 is between their reciprocals.
- **15.** Repeat problem 14 for 0.000123
- **16. •** Explain your method for solving problems 14 and 15.

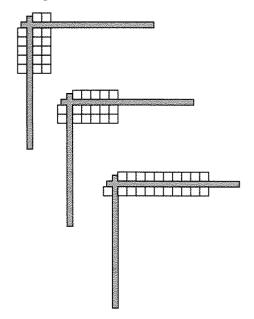
A MODEL FOR DIVISION

- 17. Exploration Find a positive number such that when you divide that number by 5, your answer is
 - a. a number less than 1;
 - b. a number between 10 and 20;
 - c. a number greater than 100.

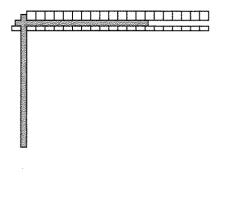
- **18.** Find a positive number such that when you divide 5 by it, your answer is
 - a. a number less than 1;
 - b. a number between 10 and 20;
 - c. a number greater than 100.

Division by numbers between 0 and 1 is hard to show with the Lab Gear.

These diagrams show 10/5, 10/2, and 10/1.



What would 10/(1/2) look like? We cannot actually build this with the Lab Gear, but we could imagine what it would look like if we sliced each block in half.



♥ 3.7

- **19.** a. What is the answer to the division shown in the figure?
 - b. Dividing by 1/2 is equivalent to multiplying by what number?
- **20.** a. Will the result of the division 8/(1/4) be more or less than 8?
 - b. Use a sketch to show the division $\frac{8}{(1/4)}$.
 - c. What is the answer to the division?
 - d. Dividing by 1/4 is equivalent to multiplying by what number?
- 21. 🐡
 - a. What is the result of the division of 8 by 0.1, 0.01, 0.001?
 - b. What would happen if you divided 8 by a number that is much smaller than 0.001, almost equal to zero?
 - c. How about dividing 8 by 0?

A DIVISION SHORTCUT

- **22.** a. If you multiplied 5 by a number and got 30, what was the number?
 - b. If you divided 5 by a number and got 30, what was the number?
 - c. Compare your answers to parts (a) and (b). How are these numbers related?

23. *Dividing by a number is the same a: multiplying by its reciprocal.* Explain, using examples.

Use this fact to perform each of the following divisions without your calculator.

24.	12/(1/4)	25. 12/(2/3)
26.	10/0.4	27. $x^2/(1/x)$

28. Find two numbers such that you get a result between 0 and 1 whether you add them, multiply them, subtract one from the other, or divide one by the other.

SMALL NUMBERS

