LESSON 3.10

Combining Functions

You will need:

graph paper

DIAGRAMS OF COMBINED FUNCTIONS

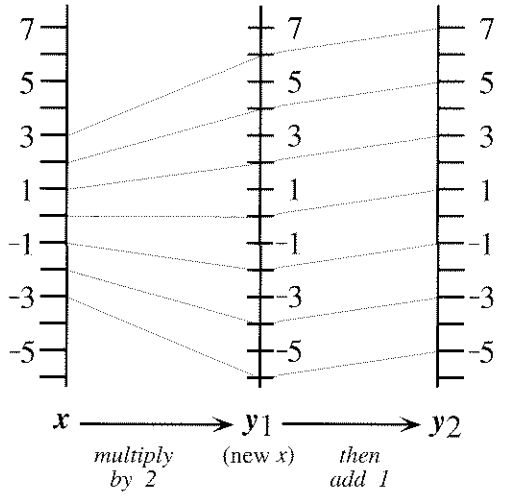
Function diagrams can be used to show the result of combining functions. Here are two simple functions. One function doubles x. The other function adds 1 to x.

$y_1 = 2x$ $\qquad$ $y_2 = x + 1$

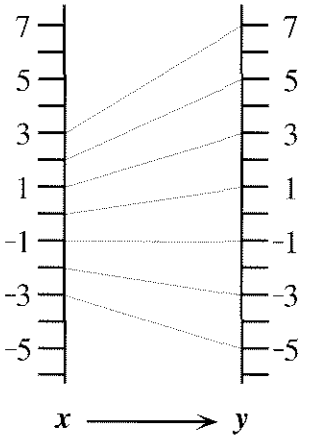
Notation: The 2 in the name y_2 is called a *subscript.* It is written lower and smaller than the y. It does *not* mean *multiply by 2* or *square.* It is just a way to distinguish two variables that would otherwise have the same name.

1. Draw function diagrams for y_1 and y_2.

This two-step function diagram shows one way of combining y_1 and y_2. First, double x. Then add 1 to the result. The y value of y_1 becomes the new x value for y_2.



These two steps can be combined as shown in this one-step function diagram.



2. Write a rule for this function diagram.

The functions y_1 and y_2 can also be combined in the other order: First, add 1 to x. Then double the result. The y value of y_2 becomes the new x value for y_1.

3. Draw a two-step function diagram showing the combination of the functions in this order.

4. Summarize your two-step function diagram in a one-step function diagram.

5. Write a rule for the one-step function diagram you drew.

6. Does the order in which we combine the functions matter? Explain.

These problems are about the following two functions.

$y_1 = -3x$ $\qquad$ $y_2 = x + 2$

7. Show a two-step function diagram, combining the functions by performing y_1 first and then y_2.

8. Summarize your two-step diagram in a one-step diagram and write the function that corresponds to your one-step function diagram.

9. Repeat problems 7 and 8, but this time combine the two functions by performing y_2 first, followed by y_1.

10. Did the resulting function change, when you changed the order in which you combined the two functions? Explain.

11. **Exploration** Sometimes you can combine two functions in either order and the resulting function is the same. Find pairs of functions that have this property. You may use function diagrams to verify your answer. Discuss any patterns you notice.

INVERSE ACTIONS

The inverse of an action is the action that undoes it. For example, suppose you were leaving home in the car. You would perform these four actions.

ACTION 1: Open the car door.
ACTION 2: Get into the car.
ACTION 3: Close the door.
ACTION 4: Start the car.

If, before driving away, you suddenly realized that you forgot something, you would have to undo all these actions. You would undo the actions in the reverse order:

First, UNDO ACTION 4: Stop the car.
Second, UNDO ACTION 3: Open the door.
Next, UNDO ACTION 2: Get out of the car.
Last, UNDO ACTION 1: Close the car door.

12. Describe how to undo these actions.

a. In the morning, you put on your socks, then put on your shoes. What do you do in the evening?

b. To take a break from this homework, you close your math book, stand up from your desk, turn on the television, and sit down on the sofa. What do you do to get back to work?

13. Al believes that the way to undo the actions *open the car window; stick your head out* is *close the car window; pull your head in.* Comment on this idea.

14. Create your own example of inverse actions.

INVERSE FUNCTIONS

15. **Exploration** Choose any function and make a function diagram for it. Then draw the mirror image of this function diagram. What is the function associated with the mirror image? How is it related to the original function? Try this with several functions. Write about any patterns you notice.

The inverse of a function is a function that undoes it. For example, look at these two input-output tables.

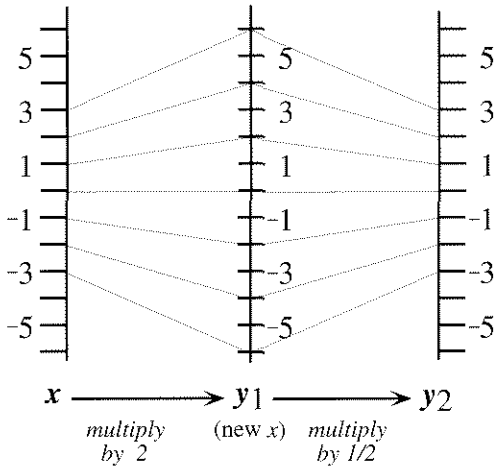
x	y
2	6
-1	-3
4	12

x	y
6	2
-3	-1
12	4

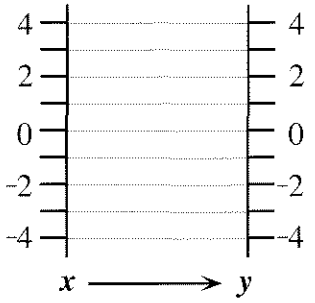
16. a. What happens when you use an output from the first table as the input for the second table?

b. What two functions do you think are represented by these two tables? How are the functions related?

If $y_1 = 2x$ and $y_2 = (1/2)x$, a two-step function diagram shows that y_2 undoes y_1.



This is shown dramatically when the two-step diagram is summarized in a one-step diagram.



A function y_1 performs the following operations on a number.

Multiply the number by 3, subtract 1.

17. Write in words what the inverse function does. (Call it y_2.)

18. a. Write a rule in the form $y_1 =$ for the original function.

b. Write a rule in the form $y_2 =$ for the inverse function.

19. a. Make separate function diagrams for y_1 and y_2.

b. Describe how the diagrams you made are related.

20. Make a two-step function diagram for the combination of y_1 and y_2.

21. Make a one-step function diagram summarizing your two-step diagram. Would it matter if you combined y_1 and y_2 in the other order?

22. Summary Write a summary of what you have learned in this lesson about combining function diagrams, especially those of inverse functions. Use examples.

23. Find functions that are their own inverses. What do you notice about their function diagrams? Explain.