Graphing Parabolas

**Definitions:**
- Second-degree polynomial functions are also called *quadratic* functions.
- Graphs of quadratic functions have a special shape called a *parabola*.
- The lowest or highest point on a parabola is called its *vertex*.

Here are two quadratic functions and their graphs. Each one has two x-intercepts and one vertex.

1. What is the y-coordinate of the x-intercepts? What is the x-coordinate of the y-intercept?
2. For each parabola in the figure,
   a. what are the x- and y-intercepts?
   b. which x-intercept is the vertex closer to?

**FINDING INTERCEPTS AND THE VERTEX**

3. a. Copy and complete the table of values for the quadratic function 
   \[ y = x^2 + 2x - 8 \]. Use at least six values from -5 to 5. Using the format shown will help you avoid making mistakes in computation.
   
<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>-4</td>
<td></td>
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<td>...</td>
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<tr>
<td>4</td>
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</tr>
<tr>
<td>5</td>
<td></td>
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</tbody>
</table>

   b. Use your table to make a graph of the function.
   c. Label the intercepts and the vertex.

4. Repeat problem 3 for the function 
   \[ y = (x + 4)(x - 2) \].

   b. How are the x-intercepts related to the expression \((x + 4)(x - 2)\)?

6. The quadratic function \[ y = x^2 - 6x + 8 \] can be written in factored form as 
   \[ y = (x - 4)(x - 2) \].
   a. Make a table of values for this function, including the intercepts and the vertex.
   b. Graph the function. Label the intercepts and the vertex.
   c. How are the x-intercepts related to the expression \((x - 4)(x - 2)\)?
   d. How is the y-intercept related to the expression \(x^2 - 6x + 8\)?
5.5

For each problem, 7-10:

a. Write the function in factored form.
b. Make a table of values, including the intercepts and the vertex.
c. Graph the function, labeling the intercepts and the vertex.

7. \( y = x^2 - 2x - 3 \)
8. \( y = x^2 + 4x + 3 \)
9. \( y = x^2 - 4x + 3 \)
10. \( y = x^2 + 2x - 3 \)

11. Write the equation of a quadratic function whose graph would cross the x-axis at (2, 0) and (-3, 0). Explain how you know it will work.

12. Write the equation of a parabola having y-intercept -4. Explain how you know it will work.

13. Consider functions of the form \( y = x^2 + bx + c \) that can be factored into \( y = (x - p)(x - q) \).
   a. How are \( b, c, p, \) and \( q \) related?
   b. How would you find the coordinates of the intercepts?
   c. How would you find the coordinates of the vertex?

SMILES AND FROWNS

14. Make a table of values for the quadratic function \( y = (x - 4)(x - 1) \) and graph it.
15. Repeat for \( y = -(x - 4)(x - 1) \).
16. Compare your graphs from problems 14-15. What is alike about the graphs and what is different? How do their x-intercepts and vertices compare?

17. Write an equation of a quadratic function whose graph satisfies these given conditions.
   a. a smile parabola having x-intercepts (3, 0) and (-2, 0)
   b. a frown parabola having x-intercepts (3, 0) and (-2, 0)
   c. a smile parabola having x-intercepts (-3, 0) and (-2, 0)
   d. a frown parabola having x-intercepts (-3, 0) and (-2, 0)

18. Explain how you know that your answers to problem 17 are correct. You may check your answers by making a table of values, and graphing.

19. Write the equation of a quadratic function that passes through the origin and (5, 0). Explain.

20. Write an equation of a quadratic function whose graph satisfies the given conditions.
   a. a parabola having one x-intercept at (1, 0) and the vertex with x-coordinate 2
   b. a parabola having one x-intercept at (1, 0) and the vertex at (2, 1)
   c. a parabola having one x-intercept at (1, 0) and the vertex at (2, 2)

HOW MANY X-INTERCEPTS?

21. Graph each of these four quadratic functions on the same axes.
   a. \( y = x^2 + 6x + 5 \)
   b. \( y = x^2 + 6x + 8 \)
   c. \( y = x^2 + 6x + 9 \)
   d. \( y = x^2 + 6x + 12 \)
22. Write a paragraph describing and comparing the graphs you drew in problem 21. Which graph or graphs have two \( x \)-intercepts? Which have one? Which have none? Could you have predicted this before graphing? Explain.

23. Consider the quadratic function \( y = x^2 + 4x + \_ \). Fill in the blank with a number that will give a function whose graph is:
   a. a parabola having one \( x \)-intercept;
   b. a parabola having two \( x \)-intercepts;
   c. a parabola having no \( x \)-intercepts.

24. Check your answers to problem 23 by graphing, or explain why you are sure you are correct.

25. Find the largest number of pennies, nickels, and dimes that you can have and still not be able to make change for a quarter. Explain your answer.

26. Find the largest number of coins you can have and still not be able to make change for a dollar. (Assume that you can have any coins except a silver dollar.) Explain this answer.

27. If \( ab = 0, bc = 0, \) and \( ac = 1 \), what is \( b \)?

28. If \( abc = 0 \) and \( bcd = 1 \), what conclusion can you draw? Explain.

29. Arrange the whole numbers from 1 to 18 into nine pairs, so that the sum of the numbers in each pair is a perfect square.