5.B Distributing

INSIDE AND OUTSIDE PRODUCTS

Look at this sequence of consecutive integers.
8, 9, 10, 11
• The product of the outside pair is 88.
• The product of the inside pair is 90.
• The difference between the inside product and the outside product is 2.

1. Find the difference between the inside and outside product for each of these sequences.
   a. 4, 5, 6, 7
   b. 10, 11, 12, 13
   c. 10, 10 + 1, 10 + 2, 10 + 3
   d. y, y + 1, y + 2, y + 3

2. What pattern did you notice in problem 1?

3. Look at some sequences of four integers that differ by three. For example, you could try 4, 7, 10, 13. What pattern do you notice in the difference between their inside and outside products?

4. What pattern would you expect to see in the difference of inside and outside products for sequences of numbers that differ by two? What about sequences of numbers that differ by four? Experiment.

5. Find the difference between the inside and outside product for each of these sequences.
   a. y, y + 2, y + 4, y + 6
   b. y, y + 3, y + 6, y + 9
   c. y, y + 5, y + 10, y + 15
   d. y, y + 5, y + 2 · 5, y + 3 · 5
   e. y, y + x, y + 2x, y + 3x

6. Write a detailed report describing the patterns you discovered in this lesson. Give examples and show all your calculations. Your report should include, but not be limited to, the answers to the following questions:

   • How is the difference between the inside and outside products related to the difference between numbers in the sequence?
   • How can you use algebra (and/or the Lab Gear) to show that your answer is correct?
   • Does your generalization work for all kinds of numbers? For example, could you choose a sequence made up entirely of negative numbers? What about fractions?

MORE DISTRIBUTIVE LAWS?

You might wonder if there are more distributive laws.

7. Is there a distributive law of exponentiation over addition? If there were, it would mean that \((x + y)^2\) would always be equal to \(x^2 + y^2\). It would also mean that \((x + y)^3\) would equal \(x^3 + y^3\). Do you think such a law exists? Explain why or why not.

8. Is there a distributive law of multiplication over multiplication? If there were, it would mean that \(a(xy)\) would always be equal to \(ax \cdot ay\). For example, \(2(xy)\) would have to equal \(2x \cdot 2y\). Do you think such a law exists? Explain why or why not.

9. Write a report about distributive laws. Use numerical examples and/or sketches of the Lab Gear. Your report should include a discussion of which of the following laws exist, and why.

   The distributive law of:
   • multiplication over addition and subtraction
   • division over addition and subtraction
   • exponentiation over addition and subtraction
   • multiplication over multiplication