LESSON 6.3
Solving Linear Equations

You will need:
the Lab Gear

1. **Exploration** Find a value of \( x \) that makes each equation true. Describe the method you used.
   a. \( 3(x + 2) = x + 5 \)
   b. \( 3x + 2 = x + 5 \)
   c. \( 3x + 2 = x - 5 \)
   d. \( 3(x + 2) = x - 5 \)

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**USING THE LAB GEAR**

The easiest equations to solve are linear, or first-degree equations in one variable. All four of the equations above are linear. The equation \( x^2 = 2x - 1 \) is not linear, because it contains an \( x^2 \) term.

You have already learned to solve equations by trial and error and the cover-up method. Some kinds of equations can also be solved using the Lab Gear.

This figure represents an equation. We want to find out what value of \( x \) will make the quantity on the left side of the workmat equal to the quantity on the right side.

2. Copy the figure with your Lab Gear.

3. Simplify each side. If you did it correctly, your blocks should match this figure.

4. Rearrange the blocks to match this figure. Which blocks on the right side can be matched with identical blocks on the left side?

There are some blocks that cannot be matched with blocks on the other side. The figure shows a two-dimensional view of these blocks.
Look at these remaining blocks. Remember that the two sides are equal. This is true even though they don’t *look* equal. Remember $x$ can have any value.

5. What must $x$ be in order for the two sides to be equal?

This figure shows how you would set out the blocks to solve the equation $2x + 1 = 4x - 5$.

![Diagram of blocks]({})

Each side is simplified. The blocks have been arranged to show which blocks can be matched with blocks on the other side. Even so, it is not easy to tell what the solution is.

It helps to add zero.

![Diagram of blocks rearranged]({})

Notice that the blocks on one side are rearranged to show which ones can be matched with blocks on the other side.

The remaining blocks (those that cannot be matched with blocks on the other side) can then be rearranged to make it easy to see the solution to the equation.

6. What is the solution to this equation?

For problems 7-11:

a. Write the original equation.

b. Use the Lab Gear to find the solution. Write equations to show some of the steps as you move your blocks.

c. Write the solution.

7. [Diagram of blocks]

8. [Diagram of blocks]
9. You have learned to solve equations using tables, trial and error, the cover-up method, and the Lab Gear. Solve these equations in whatever way you want, but show your work.

10. Make up an equation satisfying each of the following descriptions. Try to make up one that would be challenging for another student to solve, but not so challenging that you can’t solve it.

11. 12. $3x + 5 = 6$
13. $3x + 5 = -2x - 10$
14. $2y - 6 = 5y + 3$
15. $\frac{6x - 6}{4} = 3$
16. $\frac{6x - 6}{4} + 15 = 3$

17. An equation whose solution is $x = 4$

18. An equation whose solution is $y = -1/2$

19. An equation that has variables on both sides of the equation and the solution $m = 2$

20. An equation that has more than one solution


**DISCOVERY** USING VARIABLES

21. A student is \(x\) years old. How many months old is he? (Are you sure?)

22. Another student is \(y\) years old.
   a. How many years until she can vote?
   b. How many months?

23. I start with 99 peanuts. It takes me \(x\) seconds to eat one.
   a. How long will it take to eat them all?
   b. After eating \(n\) peanuts, how many are left?
   c. After \(z\) seconds, how many peanuts are left?

**PUZZLES** MAGIC SQUARES

These puzzles will be easier to solve if you make yourself little squares of paper with numbers written on them. To solve the puzzles, move the papers around, until you find a satisfactory arrangement.

24. Arrange all the numbers from 1 to 9 into a 3-by-3 square, so that the sum of all the numbers in any row or column is always the same.

25. Repeat problem 24, but make sure the diagonals also add up to the same amount.

26. Arrange all the numbers from 1 to 16 into a 4-by-4 square, so that the sum of the numbers in any row or column is always the same.

**DISCOVERY** GRADING POLICIES

At the Shell School, math teachers give a six weeks grade based on six quizzes and two writing assignments. The math department policy requires that quizzes and writing assignments be counted equally.

For each student in her class, Mrs. Washman averages the quizzes, averages the writing assignments, and then adds those two numbers and divides by two.

For each student in his class, Mr. Pitcher adds all the grades together and divides by eight.

27. Make up a list of grades of a student who would have a higher grade with Mrs. Washman’s method.

28. Make up a list of grades of a student who would have a higher grade with Mr. Pitcher’s method.

29. Is it possible for a student to have the same grade using either method? Explain.