LESSON 7.12

As the Crow Flies

You will need:

- geoboards
- dot paper

SQUARE ROOTS

As you know, the square of a number is the area of a square that has that number for a side. For example, the square of 4 is 16, because a square having side 4 has area 16.

1. a. What is the area of a square having side 9?
   b. What is the side of a square having area 9?

2. a. What is the area of a square having side 10?
   b. What is the side of a square having area 10?

You can answer question 2b with the help of a calculator, by using trial and error. Or, you may answer it by using the $\boxed{\sqrt{}}$ key.

Definition: The *square root* of a number is the side of a square that has that number for area.

For example, the square root of 4 is 2, because a square having area 4 has side 2.

3. a. What is the square of 11?
   b. What is the square root of 11?

The square root of 11 is written $\sqrt{11}$. The number given by a calculator is an approximation of the exact value. Many calculators have an $\boxed{x^2}$ key.

4. Use the $\boxed{x^2}$ key to calculate the square of 8.76. Write it down. Clear your calculator. Now use the $\boxed{\sqrt{}}$ key to find the square root of the number. What answer did you get? Explain why this is so.

5. Find a number for $\sqrt{5}$. Write it down. Now clear your calculator, enter the number, and use the $\boxed{x^2}$ key. What answer did you get? Compare your answer with other students' answers. Explain.

6. Which number has more digits, $\sqrt{10.3041}$ or $\sqrt{2}$? Make a prediction and check it with your calculator. Explain your answer.

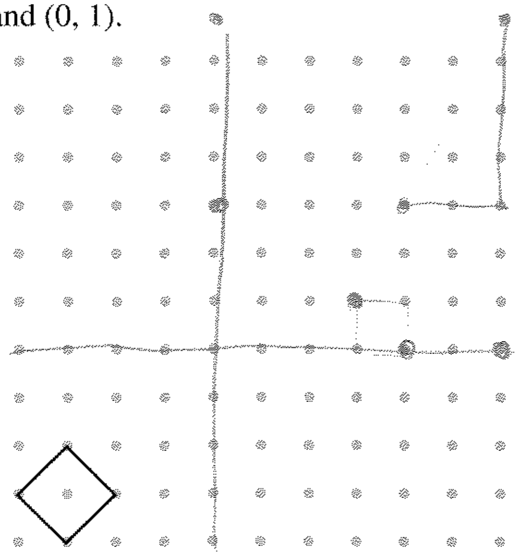
DISTANCE ON THE GEOBOARD

To find the distance between two points on the geoboard, *as the crow flies,* you can use the following strategy.

- Make a square that has the two points as consecutive vertices.
- Find the area of the square.
- Find the side of the square.

In problems 7-9, express your answers two ways: as a square root, and as a decimal approximation (unless the answer is a whole number).

Example: Find the distance between (1, 0) and (0, 1).



The area of the square is 2, so the distance between the two points is $\sqrt{2}$, or 1.41...

7. Find the distance between:
   a. (4, 3) and (6, 7);
   b. (4, 6) and (6, 4);
   c. (4, 5) and (4, 8).

8. Find the distance between the origin and (3, 1).

9. Find the distance between (5, 5) and (8, 9).

10. a. Find 12 geoboard pegs that are at a distance 5 from (5, 5). Connect them with a rubber band. Sketch the figure.
    b. Explain why someone might call that figure a *geoboard circle.*

11. How many geoboard pegs are there whose distance from (5, 5) is
    a. greater than 5?
    b. less than 5?

12. Choose a peg outside the *circle* and find its distance from (5, 5).

13. Find all the geoboard pegs whose distances from (4, 3) and (6, 7) are equal. Connect them with a rubber band. Sketch.

14. What are the distances between the pegs you found in problem 13 and (4, 3) or (6, 7)?

15. **Generalization** Describe a method for finding the distance between the origin and a point with coordinates (x, y). Use a sketch and algebraic notation.

DISCOVERY *SUMS OF PERFECT SQUARES*

16. Any whole number can be written as a sum of perfect squares. Write each whole number from 1 to 25 as a sum of squares, using *as few squares as possible* for each one. (For example, $3^2 + 1^2$ is a better answer for 10 than $2^2 + 2^2 + 1^2 + 1^2$.)

17. You should have been able to write every number in problem 16 as a sum of *four or fewer* perfect squares. Do you think this would remain possible for large numbers? For very large numbers? Experiment with a few large numbers, such as 123, or 4321.

DISCOVERY *SUMS OF POWERS*

18. Write every whole number from 1 to 30 as a sum of powers of 2. Each power of 2 cannot be used more than once for each number. Do you think this could be done with very large numbers? Try it for 100.

19. Write every whole number from 1 to 30 as a sum of powers of 3 and their opposites. Each power can appear only once for each number. Do you think this could be done with very large numbers? Try it for 100.