LESSON 8.9

Equal Powers

In this lesson, use only whole number exponents.

1. **Exploration** The number 64 can be written as a power in at least three different ways, as 2^6, 8^2, or 4^3.
   a. Find some numbers that can be written as powers in two different ways.
   b. Find another number that can be written as a power in three different ways.

POWERS OF 3 AND 9

2. Using your calculator if necessary, try to find a power of 3 that is equal to each power of 9 below. If any are impossible, say so. Fill in the exponent.
   a. $9^2 = 81 = 3^?$
   b. $9^5 = 59049 = 3^?$
   c. $9^{10} = 3^?$
   d. $9^0 = 3^?$

3. Using your calculator if necessary, try to find a power of 9 that is equal to each power of 3 below. If any are impossible, say so. Fill in the exponent.
   a. $3^8 = 6561 = 9^?$
   b. $3^5 = 243 = 9^?$
   c. $3^{14} = 9^?$
   d. $3^0 = 9^?$

4.
   a. Can every power of 9 be written as a power of 3? If so, explain why. If not, show some that can and some that can't, and explain the difference.
   b. Can every power of 3 be written as a power of 9? If so, explain why. If not, show some that can and some that can't, and explain the difference.

POWERS OF 2, 4, 6, AND 8

5. Find two powers of 2 (other than 64) that can be written as powers of 8.

6. If the same number is written as both a power of 2 and a power of 8, how do the exponents compare? Explain and give examples.

7. Find at least three powers of 2 that can be written as powers of 4. Compare the exponents and describe what you notice.

8. Find at least two powers of 2 that can be written as powers of 16. Compare the exponents and describe what you notice.

9.
   a. Which powers of 2 can be written as powers of 8? Explain, giving examples.
   b. Which powers of 8 can be written as powers of 2? Explain, giving examples.
   c. Find the smallest number (besides 1) that can be written as a power of 2, a power of 4, and a power of 8. Write it in all three ways. How do you know that it is the smallest?

10. Can you find a number that can be written as a power of 2, a power of 4, and a power of 6? If so, find it. If not, explain why it is impossible.

WRITING POWERS USING DIFFERENT BASES

11. Write each number as a power using a smaller base.

a. 8^2	b. 27^3	c. 25^3
d. 16^4	e. 49^2	f. 2^0

12. Write each number as a power using a larger base.

a. 3^2 b. 9^4 c. 4^8
d. 5^8 e. 6^6 f. 95^0

13. If possible, write each number as a power using a different base. (Do not use the exponent 1.) If it is not possible, explain why not.

a. 3^4 b. 3^3
c. 4^5 d. 3^5

14. Repeat problem 13 for these numbers.

a. 5^4 b. 5^3
c. 25^2 d. 26^4

15. Summary If you exclude the exponent 1, when it is possible to write a number in two or more ways as a power? Does it depend on the base, the exponent, or both? Explain. (Give examples of some equivalent powers and of numbers that can be written as powers in only one way.)

16. Generalization Fill in the exponents.

a. $9^x = 3^?$ b. $4^x = 2^?$
c. $8^x = 2^?$ d. $16^x =$
e. $25^x =$

A POWER OF A POWER

Since $9 = 3^2$, the power 9^3 can be written as $(3^2)^3$. The expression $(3^2)^3$ is a *power of a power* of 3.

17. a. Write 25^3 as a power of a power of 5.
b. Write 8^5 as a power of a power of 2.
c. Write 9^4 as a power of a power of 3.

There is often a simpler way to write a power of a power. For example:

$$\begin{aligned}(3^5)^2 &= (3^5)(3^5)\\ &= (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)\\ &= 3^{10}\end{aligned}$$

18. a. Show how $(2^5)^3$ can be written with one exponent as a power of 2.
b. Write $(3^4)^2$ as a power of 3.

19. Is $(4^5)^3$ equal to 4^8, to 4^{15}, or to neither? Explain.

Generalizations

20. Fill in the exponents.

a. $(x^2)^3 = x^?$ b. $y^4 = (y^2)^?$
c. $y^{10} = (y^5)^?$ d. $y^6 = (y^3)^?$
e. $(x^4)^3 = x^?$

21. Fill in the exponents.

a. $(y^2)^x = y^?$ b. $(y^3)^x = y^?$
c. $(x^4)^y = x^?$ d. $y^{ax} = (y^x)^?$

The generalization you made in problem 21 is one of the *laws of exponents.* It is sometimes called the *power of a power law:*

$$(x^a)^b = x^{ab}, \text{ as long as } x \text{ is not } 0.$$

22. Explain how the ideas you discussed in problem 15 are related to the power of a power law.