LESSON 9.3

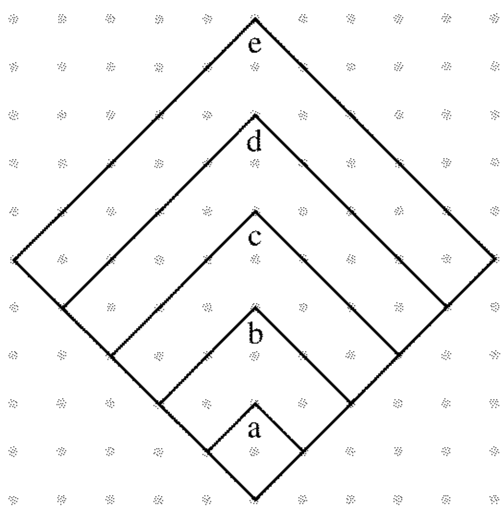
Radicals

You will need:

geoboards

dot paper

SQUARES AND ROOTS



The figure shows five squares. For each one, find

1. its area;
2. its side, written twice: as the square root of the area, and as a decimal number.

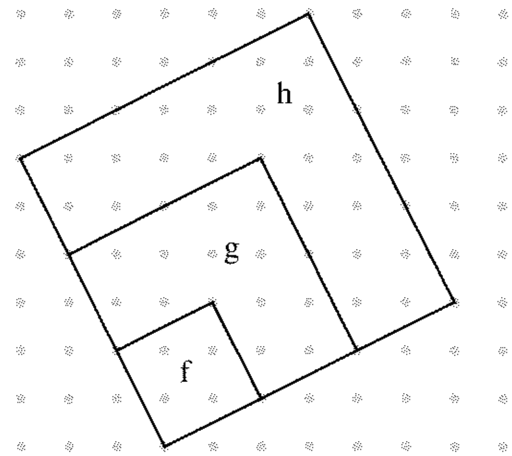
The sides of the larger squares are multiples of the side of the smallest square. For example, square (b) has a side that is equal to two times the side of square (a). You can write,

$$\sqrt{8} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}.$$

Note that $2\sqrt{2}$ means 2 times $\sqrt{2}$, just as $2x$ means 2 times x. You can check the equation with a calculator.

$$\sqrt{8} = 2.828427125\ldots$$
$$2\sqrt{2} = 2.828427125\ldots$$

3. Write equations about the sides of squares (c), (d), and (e). Check their correctness with a calculator.



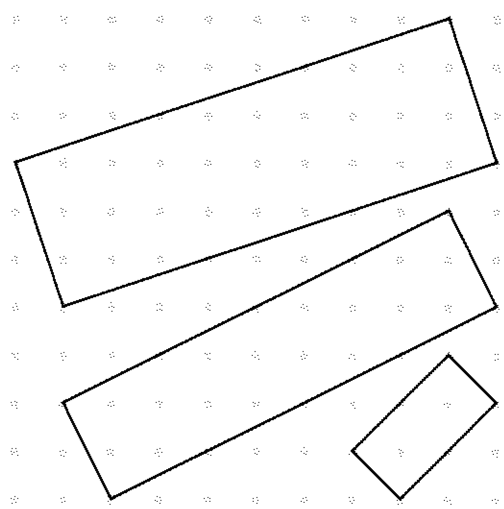
The figure shows three squares. For each one, find

4. its area;
5. its side, written twice: as the square root of the area, and as a decimal number.
6. Write equations involving square roots based on the figure. Check your equations on a calculator.
7. True or False? Use a sketch on dot paper to explain your answers.
   a. $\sqrt{2} + \sqrt{2} = \sqrt{4}$
   b. $4\sqrt{2} = \sqrt{8}$
8. Is $\sqrt{2 + 2} = \sqrt{4}$? Explain.

RECTANGLES AND ROOTS

In this section do not use decimal approximations.

9. The figure shows three rectangles. For each one, write *length* · *width* = *area*.

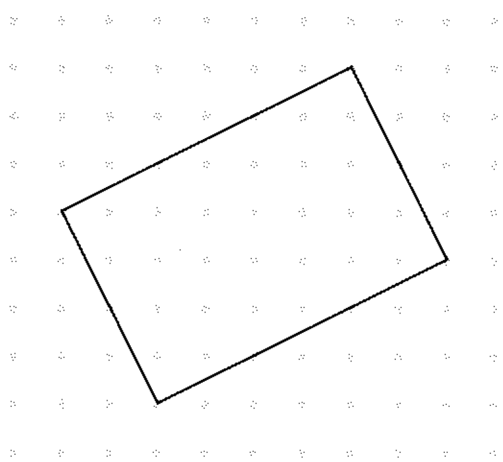


10. For each rectangle above:

a. What is the side of a square having the same area?

b. Sketch this square on dot paper.

Some multiplications involving square roots can be modeled by geoboard rectangles. For example, $2\sqrt{5} \cdot 3\sqrt{5}$ is shown in this figure.



11. Find the product of $2\sqrt{5} \cdot 3\sqrt{5}$ by finding the area of the rectangle.

12. Multiply.

a. $2\sqrt{2} \cdot 3\sqrt{2}$ b. $3\sqrt{2} \cdot 4\sqrt{2}$

c. $4\sqrt{2} \cdot 5\sqrt{2}$ d. $\sqrt{2} \cdot 2\sqrt{2}$

13. Multiply.

a. $\sqrt{2} \cdot \sqrt{18}$ b. $\sqrt{18} \cdot \sqrt{50}$

c. $\sqrt{50} \cdot \sqrt{8}$ d. $\sqrt{8} \cdot \sqrt{32}$

Using the fact that $\sqrt{a} \cdot \sqrt{a} = a$ makes it easy to multiply some quantities involving radicals. For example:

$$6\sqrt{5} \cdot 2\sqrt{5} = 6 \cdot 2 \cdot \sqrt{5} \cdot \sqrt{5} = 12 \cdot 5 = 60$$

14. Multiply.

a. $5\sqrt{2} \cdot \sqrt{2}$ b. $5\sqrt{2} \cdot 4\sqrt{2}$

c. $3\sqrt{5} \cdot \sqrt{5}$

15. Explain your answers by using a sketch of a geoboard rectangle.

a. Is $\sqrt{4} \cdot \sqrt{2} = \sqrt{8}$?

b. Is $\sqrt{5} \cdot \sqrt{20} = \sqrt{100}$?

MULTIPLYING SQUARE ROOTS

Is it always true that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$? We cannot answer this question in general by making geoboard rectangles. A multiplication like $\sqrt{2} \cdot \sqrt{5}$ cannot be shown that way because it is not possible to find those lengths on the geoboard at a right angle to each other.

16. Guess how to write $\sqrt{2} \cdot \sqrt{5}$ as a square root. Check your guess with a calculator.

17. **Generalization** If a and b are positive,

a. give a rule for multiplying $\sqrt{a} \cdot \sqrt{b}$;

b. explain how to multiply $c\sqrt{a} \cdot d\sqrt{b}$.

18. Multiply.

a. $3\sqrt{5} \cdot 2\sqrt{6}$

b. $(2\sqrt{11})(-11\sqrt{2})$

SIMPLE RADICAL FORM

Definitions: The square root symbol ($\sqrt{\ }$) is called a *radical sign,* or simply *radical.* A *radical expression* is an expression that includes a radical.

Examples:

$$\sqrt{3},\ 4\sqrt{7},\ 1 + \sqrt{6}, \text{ or } \frac{\sqrt{2}}{x}$$

19. Write each of these in at least two ways as the product of two radical expressions.

a. $\sqrt{70}$ b. $\sqrt{63}$

c. $6\sqrt{80}$ d. $24\sqrt{105}$

20. Write each of these as the product of two radicals, one of which is the square root of a perfect square.

a. $\sqrt{75}$ b. $\sqrt{45}$
c. $\sqrt{98}$ d. $\sqrt{28}$

Definition: Writing the square root of a whole number as a product of a whole number and the square root of a smallest possible whole number is called putting it in *simple radical form.*

For example, in simple radical form,
$\sqrt{50}$ is $5\sqrt{2}$ $\sqrt{20}$ is $2\sqrt{5}$.

(Note that when using a calculator to find an approximate value, simple radical form is not simpler!)

21. Write in simple radical form.

a. $\sqrt{75}$ b. $\sqrt{45}$
c. $\sqrt{98}$ d. $\sqrt{28}$

GEOBOARD LENGTHS

Since 50 is a little more than 49, $\sqrt{50}$ is a little more than 7. A calculator confirms this: $\sqrt{50} = 7.07\ldots$

22. Estimate the following numbers, and check your answer on a calculator.

a. $\sqrt{65}$ b. $\sqrt{85}$

These numbers may help you with the next problem.

23. Exploration There are 19 geoboard line segments that start at the origin and have length 5, 10, $\sqrt{50}$, $\sqrt{65}$, or $\sqrt{85}$. Find them, and mark their endpoints on dot paper.

24. If you know two sides of a geoboard triangle are of length 5, what are the possibilities for length for the third side?

25. Repeat problem 24 for the following side lengths.

a. 10 b. $\sqrt{50}$
c. $\sqrt{65}$ d. $\sqrt{85}$