LESSON 9.3 Radicals

You will need:
- geoboards

The figure shows five squares. For each one, find
1. its area;
2. its side, written twice: as the square root of the area, and as a decimal number.

The sides of the larger squares are multiples of the side of the smallest square. For example, square (b) has a side that is equal to two times the side of square (a). You can write,
\[ \sqrt{8} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}. \]
Note that \( 2\sqrt{2} \) means 2 times \( \sqrt{2} \), just as \( 2x \) means 2 times \( x \). You can check the equation with a calculator.
\[ \sqrt{8} \approx 2.828427125 \ldots \]
\[ 2\sqrt{2} \approx 2.828427125 \ldots \]

3. Write equations about the sides of squares (c), (d), and (e). Check their correctness with a calculator.

The figure shows three squares. For each one, find
4. its area;
5. its side, written twice: as the square root of the area, and as a decimal number.
6. Write equations involving square roots based on the figure. Check your equations on a calculator.

7. True or False? Use a sketch on dot paper to explain your answers.
   a. \( \sqrt{2} + \sqrt{2} = \sqrt{4} \)
   b. \( 4\sqrt{2} = \sqrt{8} \)

8. Is \( \sqrt{2} + 2 = \sqrt{4} \)? Explain.

In this section do not use decimal approximations.

The figure shows three rectangles. For each one, write \( \text{length} \cdot \text{width} = \text{area} \).

Chapter 9 Measurement and Square Roots
10. For each rectangle above:
   a. What is the side of a square having
      the same area?
   b. Sketch this square on dot paper.

Some multiplications involving square roots
   can be modeled by geoboard rectangles. For
   example, \(2\sqrt{5} \cdot 3\sqrt{5}\) is shown in this figure.

11. Find the product of \(2\sqrt{5} \cdot 3\sqrt{5}\) by finding
   the area of the rectangle.

12. Multiply.
   a. \(2\sqrt{2} \cdot 3\sqrt{2}\)
   b. \(3\sqrt{2} \cdot 4\sqrt{2}\)
   c. \(4\sqrt{2} \cdot 5\sqrt{2}\)
   d. \(\sqrt{2} \cdot 2\sqrt{2}\)

   a. \(\sqrt{2} \cdot \sqrt{18}\)
   b. \(\sqrt{18} \cdot \sqrt{50}\)
   c. \(\sqrt{50} \cdot \sqrt{8}\)
   d. \(\sqrt{8} \cdot \sqrt{32}\)

   a. \(5\sqrt{2} \cdot \sqrt{2}\)
   b. \(5\sqrt{2} \cdot 4\sqrt{2}\)
   c. \(3\sqrt{5} \cdot \sqrt{5}\)

15. Explain your answers by using a
    sketch of a geoboard rectangle.
    a. Is \(\sqrt{4} \cdot \sqrt{2} = \sqrt{8}\)?
    b. Is \(\sqrt{5} \cdot \sqrt{20} = \sqrt{100}\)?

16. Guess how to write \(\sqrt{2} \cdot \sqrt{5}\) as a square
    root. Check your guess with a calculator.

17. Generalization
    If \(a\) and \(b\) are positive,
    a. give a rule for multiplying \(\sqrt{a} \cdot \sqrt{b}\);
    b. explain how to multiply \(c\sqrt{a} \cdot d\sqrt{b}\).

18. Multiply.
   a. \(3\sqrt{5} \cdot 2\sqrt{6}\)
   b. \((2\sqrt{11})(-11\sqrt{2})\)

19. Write each of these in at least two ways as
    the product of two radical expressions.
   a. \(\sqrt{70}\)
   b. \(\sqrt{63}\)
   c. \(6\sqrt{80}\)
   d. \(24\sqrt{105}\)

Using the fact that \(\sqrt{a} \cdot \sqrt{a} = a\) makes it easy
   to multiply some quantities involving radicals.
   For example:
   \[6\sqrt{5} \cdot 2\sqrt{5} = 6 \cdot 2 \cdot \sqrt{5} \cdot \sqrt{5} = 12 \cdot 5 = 60\]
20. Write each of these as the product of two radicals, one of which is the square root of a perfect square.
   a. \( \sqrt{75} \)  
   b. \( \sqrt{45} \)  
   c. \( \sqrt{98} \)  
   d. \( \sqrt{28} \)

**Definition:** Writing the square root of a whole number as a product of a whole number and the square root of a smallest possible whole number is called putting it in *simple radical form.*

For example, in simple radical form, 
\[ \sqrt{50} = 5\sqrt{2} \quad \sqrt{20} = 2\sqrt{5} \]
(Note that when using a calculator to find an approximate value, simple radical form is not simpler!)

21. Write in simple radical form.
   a. \( \sqrt{75} \)  
   b. \( \sqrt{45} \)  
   c. \( \sqrt{98} \)  
   d. \( \sqrt{28} \)

22. Estimate the following numbers, and check your answer on a calculator.
   a. \( \sqrt{65} \)  
   b. \( \sqrt{85} \)

These numbers may help you with the next problem.

23. **Exploration** There are 19 geoboard line segments that start at the origin and have length 5, 10, \( \sqrt{50}, \sqrt{65}, \) or \( \sqrt{85} \). Find them, and mark their endpoints on dot paper.

24. If you know two sides of a geoboard triangle are of length 5, what are the possibilities for length for the third side?

25. Repeat problem 24 for the following side lengths.
   a. 10  
   b. \( \sqrt{50} \)  
   c. \( \sqrt{65} \)  
   d. \( \sqrt{85} \)