LESSON 9.4

Radical Operations

You will need:

geoboards

dot paper

MULTIPLICATION

1. **Exploration** Using only multiplication, write at least three radical expressions that equal each of the following.

   a. $2\sqrt{3}$ b. 6

Even though you are often asked to simplify expressions, it is sometimes just as important to know how to "complicate" them. For example, $3\sqrt{7}$ is equivalent to all these radical expressions.

$\sqrt{9}\sqrt{7}$ $\sqrt{9 \cdot 7}$ $\sqrt{63}$

$\sqrt{3}\sqrt{3}\sqrt{7}$ $\sqrt{3}\ \sqrt{21}$

2. Write at least two other radical expressions equivalent to:

   a. $5\sqrt{2}$; b. $2\sqrt{5}$;
   c. $6\sqrt{10}$; d. $10\sqrt{6}$.

3. Write each as the square root of a number. (For example, $3\sqrt{7} = \sqrt{63}$.)

   a. $2\sqrt{2}$ b. $2\sqrt{7}$
   c. $5\sqrt{6}$ d. $4\sqrt{3}$

4. Write each as the product of as many square roots as possible. (For example, $3\sqrt{6} = \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{3}$.)

   a. $5\sqrt{10}$ b. $7\sqrt{5}$
   c. $\sqrt{30}$ d. $10\sqrt{22}$

5. What number times $\sqrt{6}$ equals $3\sqrt{10}$?

To answer problem 5, Tina wrote:

$\_\_\_\_\_ \cdot \sqrt{6} = 3\sqrt{10}$

$\_\_\_\_\_ \cdot \sqrt{3}\sqrt{2} = \sqrt{3}\sqrt{3}\sqrt{2}\sqrt{5}$

"First I wrote everything as a product of square roots," she explained. "Then it was easy to see that the missing factors were $\sqrt{5}$ and $\sqrt{3}$, so the answer must be $\sqrt{15}$."

Erin politely told Tina that her method seemed unnecessarily complicated. Erin wrote:

$\_\_\_\_\_ \cdot \sqrt{6} = 3\sqrt{10}$

$\_\_\_\_\_ \cdot \sqrt{6} = \sqrt{9}\sqrt{10}$

$\_\_\_\_\_ \cdot \sqrt{6} = \sqrt{90}$

"My goal was to write $3\sqrt{10}$ as the square root of something. Once I found that $3\sqrt{10} = \sqrt{90}$, it was easy from there. I could use the rule that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ to see that the answer was $\sqrt{15}$," she explained.

6. What number times $2\sqrt{10}$ equals $10\sqrt{2}$? Find the answer by using

   a. Tina's method;
   b. Erin's method.

7. What number times $\sqrt{8}$ equals 4?

8. What number times $2\sqrt{2}$ equals $4\sqrt{3}$?

DIVISION

9. Divide 5 by $2\sqrt{5}$.

"That's not fair," said Tina. "Ms. Kem never taught us to divide with radicals." "That's true," said Erin, "but we know that multiplication and division are inverse operations." She wrote:

$\_\_\_\_\_ \cdot 2\sqrt{5} = 5$

$\_\_\_\_\_ \cdot \sqrt{4}\sqrt{5} = \sqrt{25}$

10. Finish solving the problem using Erin's method.

Another way to solve this problem is to use the following trick: *Write an equivalent fraction without a square root in the denominator.* In this case, we multiply both the numerator and denominator by $\sqrt{5}$.

$$\frac{5}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5 \cdot \sqrt{5}}{2 \cdot 5} = \frac{\sqrt{5}}{2}$$

11. Explain why $\sqrt{5}$ was chosen as the number by which to multiply.

12. Divide.

a. $\frac{3}{2\sqrt{6}}$ b. $\frac{24}{\sqrt{6}}$ c. $\frac{3\sqrt{10}}{5\sqrt{3}}$ d. $\frac{5\sqrt{3}}{3\sqrt{10}}$

MORE ON SIMPLE RADICAL FORM

Using the fact that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$, we can write $\sqrt{63}$ as $\sqrt{21} \cdot \sqrt{3}$. We can also write it as $\sqrt{9} \cdot \sqrt{7}$, which is especially convenient because 9 is a perfect square. Therefore:

$$\sqrt{63} = \sqrt{9} \cdot \sqrt{7} = 3\sqrt{7}.$$

This last expression is in simplest radical form.

13. Write in simple radical form.

a. $\sqrt{200}$ b. $\sqrt{147}$

c. $\sqrt{700}$ d. $\sqrt{275}$

ADDITION AND SUBTRACTION

14. Use dot paper to illustrate the addition $\sqrt{5} + 2\sqrt{5}$.

15. Using the figure you made in problem 14, explain how to decide which of the two equations $\sqrt{5} + \sqrt{20} = \sqrt{25}$ and $\sqrt{5} + \sqrt{20} = \sqrt{45}$ is correct.

16. Check your answer to problem 15 with a calculator.

17. True or False? Explain.

a. $16 + 9 = 25$

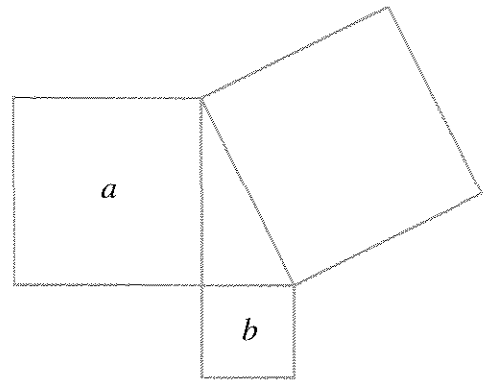
b. $\sqrt{16 + 9} = \sqrt{25}$

c. $\sqrt{16} + \sqrt{9} = \sqrt{25}$

d. $\sqrt{16} + \sqrt{9} = \sqrt{16 + 9}$

18. If a and b are positive numbers, is it always, sometimes, or never true that $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$? Explain, with examples.

The figure shows a right triangle, a square having area a, a square having area b, and a third square.



19. In terms of a and b,

a. what is the area of the third square? Explain.

b. What are the sides of the triangle?

20. If a and b are positive numbers, is it always, sometimes, or never true that $\sqrt{a} + \sqrt{b} > \sqrt{a + b}$? Explain, using the figure.

As you see, sums of radical expressions cannot usually be simplified. However, in some cases, simple radical form can help.

21. Simplify, then add or subtract.

a. $\sqrt{18} + \sqrt{32}$

b. $\sqrt{18} - 4\sqrt{20}$

c. $\sqrt{60} - \sqrt{135}$

d. $\sqrt{45} + \sqrt{125}$

You can add or subtract square roots only if they are the roots of the same number. This is similar to combining like terms when adding polynomials.

22. Simplify, then add or subtract, if possible.

a. $5 + 5\sqrt{68} + \sqrt{17}$

b. $6 - 6\sqrt{15} + \sqrt{90}$

c. $\sqrt{8} + \sqrt{16} + \sqrt{32} - \sqrt{64}$

d. $\sqrt{10} + \sqrt{20} - \sqrt{30} + \sqrt{40} - \sqrt{50}$