LESSON 10.4

Solving Systems

You will need:

the Lab Gear 

Definitions: In real-world applications we often need to find a solution that satisfies two or more equations simultaneously. We call the group of equations a *system of simultaneous equations*. To *solve a system* means to find the (x, y) pairs that satisfy every equation in the group.

In this course, you will learn techniques for solving systems of two equations. In later courses you will learn how to solve systems of more than two equations.

In an earlier chapter, you studied equivalent equations. *Equivalent equations have all the same solutions.*

1. Find some (x, y) solutions to these equations.

a. $y = 2x + 6$

b. $3y = 6x + 18$

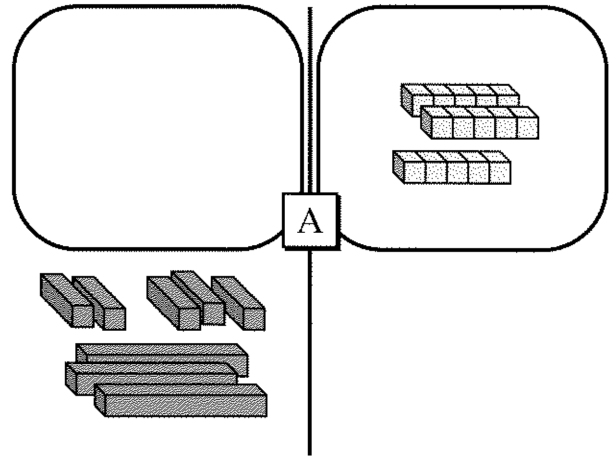
2. Use algebra to show that the two equations in problem 1 are equivalent.

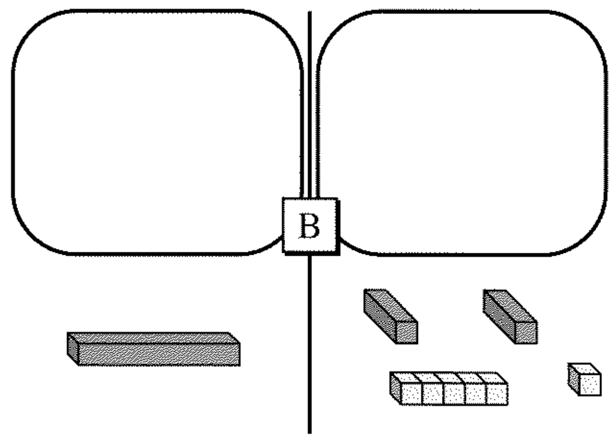
SOLVING TECHNIQUES: SUBSTITUTION

Example: Solve the system.

$$\begin{cases} 5x + 3y = -15 & \text{(A)} \\ y = 2x + 6 & \text{(B)} \end{cases}$$

The figure shows how to model the system on two workmats.

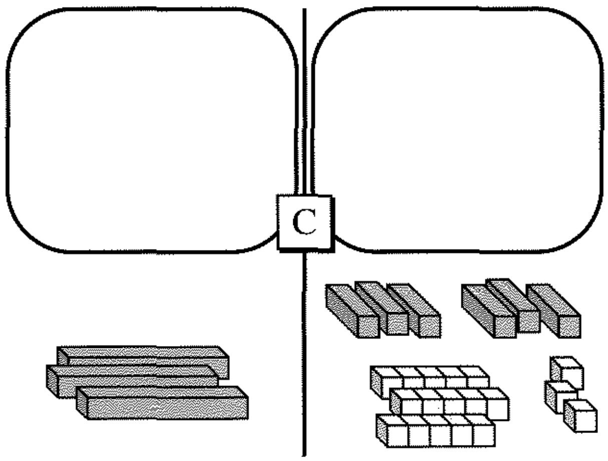




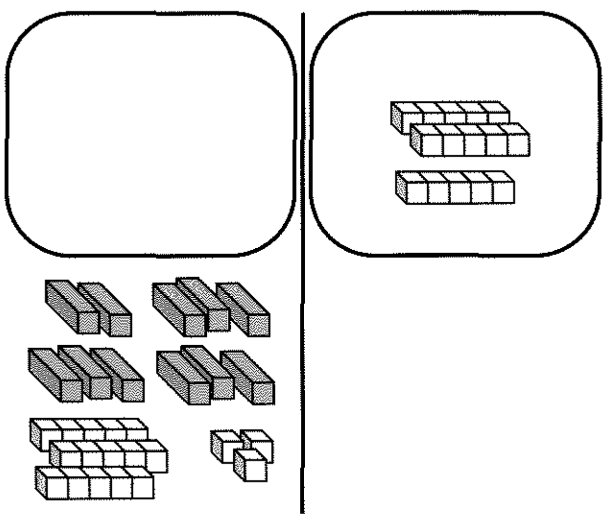
By multiplying both sides of Equation (B) by 3, you get Equation (C), which is equivalent to Equation B.

$$3y = 6x + 18 \quad \text{(C)}$$

The figure shows how to model Equation (C) on the second workmat.



Since $3y = 6x + 18$, we can replace the $3y$ in the first equation with $6x + 18$ to get a new equation that has only x-blocks and yellow blocks.



3. Write the new equation. Then solve for x.
4. a. Substitute the value of x into Equation (B) and solve for y.
   b. Substitute this (x, y) pair into Equation (A). If it doesn't satisfy the equation, check your work to find your mistake.
   c. Write the (x, y) pair that is the solution to the system.

Solve each system, 5-10. If you use the Lab Gear, you may set up the first equation with the blocks. Then use the second equation to eliminate the x- or y-blocks by substitution. In some cases, you may first need to write an equation equivalent to the second equation.

5. $\begin{cases} 5y - 4x = -9 \\ 5y = 3x - 7 \end{cases}$ **6.** $\begin{cases} 5x + 3y = -15 \\ y = 2x + 6 \end{cases}$

7. $\begin{cases} 5x - 3y = -29 \\ x = 2 - 2y \end{cases}$ **8.** $\begin{cases} 2x + 3y = 9 \\ 4x = 6 - 2y \end{cases}$

9. $\begin{cases} 4x - y = 5 \\ 3y = 6x + 3 \end{cases}$ **10.** $\begin{cases} 6x - 2y = -16 \\ 4x + y = 1 \end{cases}$

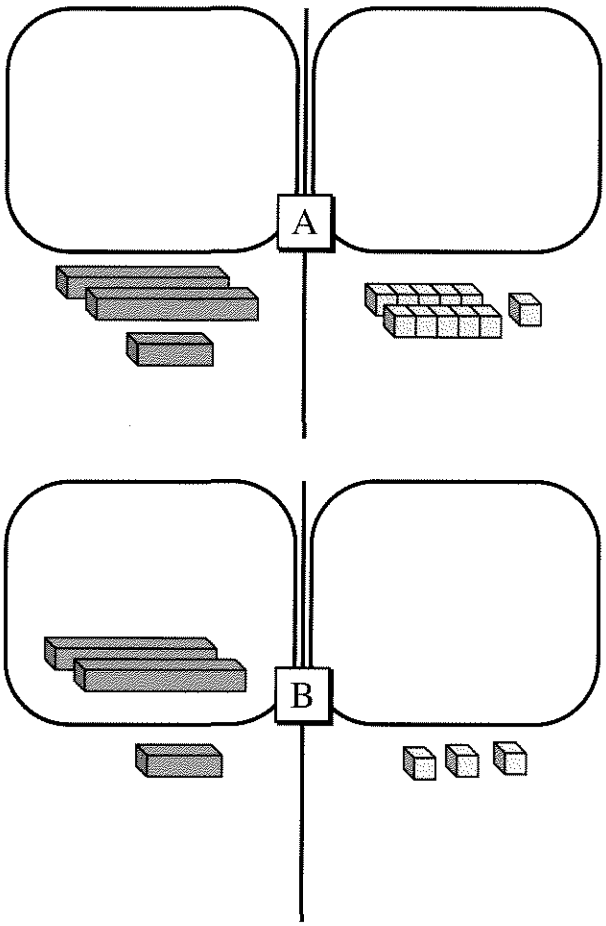
SOLVING TECHNIQUES: LINEAR COMBINATIONS

Here is another technique for solving systems.

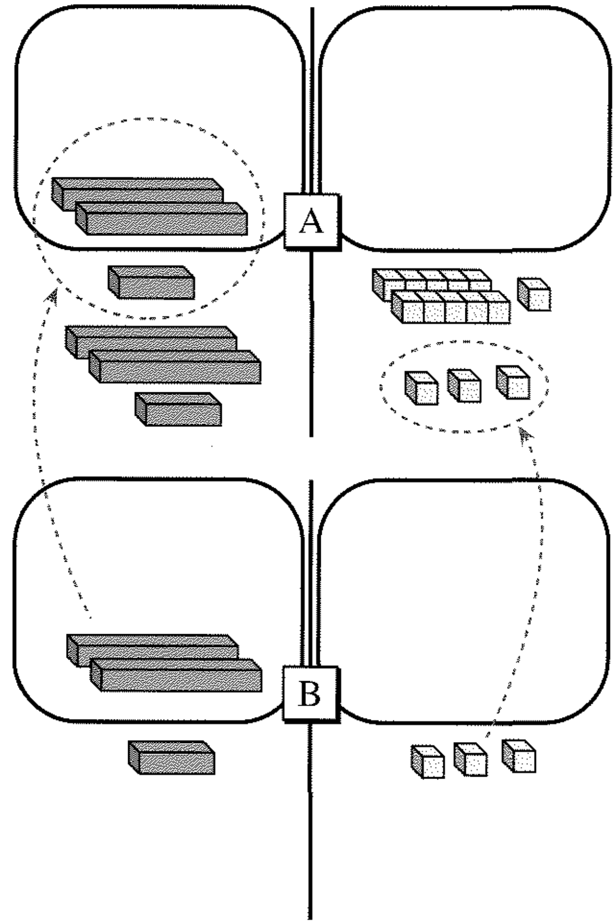
Example: Solve the system.

$$\begin{cases} x + 2y = 11 & \text{(A)} \\ x - 2y = 3 & \text{(B)} \end{cases}$$

The figure shows two workmats, with one equation modeled on each.



You can add equal quantities to both sides of Equation (A) to get an equivalent equation. For example, you could add 3 to both sides, or even $x - 2y$ to both sides. Also, since Equation (B) says that $x - 2y = 3$, you could add 3 to one side and $x - 2y$ to the other side, as shown on the figure.



11. Write the equation shown in equation (A) in the figure. Simplify and solve for x. (What happened to y?)

12. Find the (x, y) pair that is the solution to the system. Check by substituting into both of the original equations.

Solving the system in the example was easier than solving most systems, since when you added one equation to the other there were no y's left. The next example is more difficult.

Example: Solve the system.

$$\begin{cases} 2y - 6x = 16 & \text{(A)} \\ 4x + y = 1 & \text{(B)} \end{cases}$$

By multiplying both sides of Equation (B) by -2, you get Equation (C), which is equivalent to Equation (B). Here is the new system, which is equivalent to the original.

$$\begin{cases} 2y - 6x = 16 & \text{(A)} \\ -8x - 2y = -2 & \text{(C)} \end{cases}$$

13. Why was Equation (B) multiplied by -2?

14. Solve the system. Show your work. Check your answers by substituting into both equations of the original system.

Mr. Richards gave the class this hard system to solve.

$$\begin{cases} 3x + 5y = 17 & \text{(A)} \\ 2x + 3y = 11 & \text{(B)} \end{cases}$$

Charlotte suggested multiplying the first equation by 3 and the second equation by -5 to get a new system.

15. Use Charlotte's method to write a new system. Solve the system and check your answer.

Definition: The equation you get by adding multiples of the two equations together is called a *linear combination* of the two equations.

Leroy thought it would be easier if they got a linear combination by multiplying by smaller numbers. He suggested multiplying the first equation by -2 and the second equation by 3.

16. Use Leroy's method to write a new system. Solve the system.

17. Compare the two ways you solved this problem. Which do you prefer? Can you think of a third way? Explain.

SYSTEMATIC PRACTICE

Solve these systems. Some have one (x, y) solution. Others have an infinite number of solutions, or no solution.

18. $\begin{cases} 5x + 7y = 1 \\ x + 7 = 1 \end{cases}$

19. $\begin{cases} 3 - x = 4y \\ x = -2y - 9 \end{cases}$

20. $\begin{cases} 8x - 4y = 0 \\ 2x = y \end{cases}$

21. $\begin{cases} y = 4 + x \\ y = 7x + 10 \end{cases}$

22. $\begin{cases} 4x - y = 2 \\ y = 4x + 1 \end{cases}$

23. $\begin{cases} 6x - 2y = -16 \\ 4x + y = 1 \end{cases}$