LESSON 11.7

Random Walks

You will need:

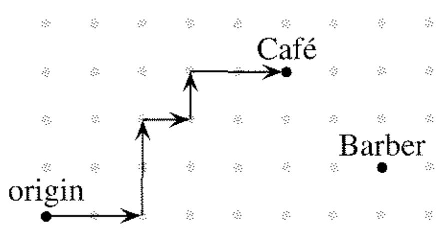
dot paper

coins
(pennies, nickels, dimes, quarters)

The Mad Probabilist takes a random walk on dot paper. Starting at the origin, he goes from lattice point to lattice point, flipping a coin each time to determine where to go next.

- *Heads* means to move east, increasing just the x-coordinate by 1.
- *Tails* means to move north, increasing just the y-coordinate by 1.

The map shows the path H, H, T, T, H, T, H, H.



1. **Exploration** Find another sequence of heads and tails that would get the Mad Probabilist from the origin to (5, 3), where the Slow Food Café is located. Compare your sequence with that of a classmate. How many ways are there to reach (5, 3)?

A FOUR-COIN EXPERIMENT

2. If you toss a penny, a nickel, a dime, and a quarter, which do you think is most likely to occur: 0 heads, 1 head, 2 heads, 3 heads, 4 heads? Or are they all equally likely? Explain your reasoning.

3. Use a penny, a nickel, a dime, and a quarter. Toss them and record the number of heads. Repeat this experiment 20 times.

If you toss a penny, a nickel, a dime, and a quarter, the event *three heads* consists of the following equally likely outcomes: HHHT, HHTH, HTHH, and THHH, depending on which coin comes up tails.

4. Find all possible equally likely outcomes when tossing four coins.

5. Count the outcomes for each of these events: 0 heads, 1 head, 2 heads, etc.

6. Are the results of your experiment in problem 3 consistent with your analysis in problems 4 and 5? Comment.

If you toss one coin, there are two equally likely possible outcomes, H and T. In Lesson 6 you studied the tossing of two coins, (HH, HT, TH, TT), and in problems 5-6 the tossing of four coins.

7. Figure out how many equally likely outcomes are possible if you toss
a. three coins; b. five coins.

8. **Generalization** How many equally likely outcomes are possible if you toss n coins? Explain.

Tossing the same coin repeatedly works in a similar way. For example, one possible string of eight tosses is: TTHTHTTH, just as one possible outcome of tossing eight coins is TTHTHTTH.

9. If you toss one coin eight times, how many possible outcomes are there? How about n times?

THE MAD PROBABILIST

10. How many moves does it take the Mad Probabilist to get to (5, 3)?

11. Generalization How many moves does it take him to get to (p, q)? Explain.

12. a. Where might he be after six moves?
b. Make a list of the points he could get to in seven moves.

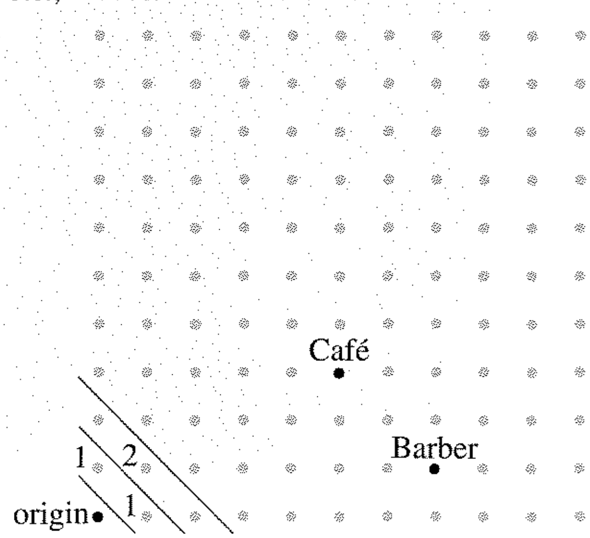
13. How would you describe the set of points you listed in problem 12b? (How many points does it consist of? What equation relates their coordinates?) Explain.

14. Generalization Describe the set of points he could reach in n moves. Explain.

15. Which is greater, the number of possible points he could end up on after eight tosses of a coin, or the number of possible strings of eight tosses? Explain.

MAKING A MAP

The Mad Probabilist wants to calculate the *probability* of getting to a lattice point like (5, 3). He decides to make a map on a piece of dot paper. He draws diagonal lines to separate the points he may reach in one, two, three, etc., moves.



Then he writes how many ways there are to reach each point on the map. For example, there is only one way to get to (1, 0): a toss of H. There is only one way to get to (0, 1): T. There is only one way to get to (2, 0): HH. There are two ways to get to (1, 1): HT or TH.

As he makes his map, he finds it helpful to ask himself for each point, "Where could I have come from to get here?"

16. Continue the Mad Probabilist's map, until you get to (5, 3).

The Mad Probabilist reasons, "At the end of eight moves, I will be at one of these points, one of which is the Slow Food Café." He marks the points on his map. "The outcomes are eight-move paths; the event is those paths that end up at (5, 3). To find out the probability of this event, I need a numerator and a denominator." He writes:

$$P(5, 3) = \frac{\text{\# of paths that get to (5, 3)}}{\text{\# of 8-move paths}}$$

17. What is $P(5, 3)$? In other words, what is the probability the Mad Probabilist's random walk will end up at the Slow Food Café?

18. What is the probability it will end up at (7, 1), where the barbershop is? Explain.

19. Summary Explain how you can find the probability of getting to any lattice point in the first quadrant.

DISCOVERY ***PASCAL PATTERNS***

This is one of the most important arrays of numbers in mathematics. It is called Pascal's triangle.

1								
1	1							
1	2	1						
1	3	3	1					
1	4	6	4	1				
1	5	10	10	5	1			
1	6	15	20	15	6	1		
1	7	21	35	35	21	7	1	
1	8	28	56	70	56	28	8	1

20. **Exploration** Study this triangle, looking for patterns. Explain any patterns that you find.

21. Find a pattern that will enable you to write the next row in the triangle.

22. Find the pattern in the third column.

23. Find the pattern in the sums of the rows.

24. Find the pattern in the sums of the upward diagonals.

1								
1	1							
1	2	1						
1	3	3	1					
1	4	6	4	1				
1	5	10	10	5	1			
1	6	15	20	15	6	1		
1	7	21	35	35	21	7	1	
1	8	28	56	70	56	28	8	1

25. **Report** Write an illustrated report about the patterns you found in Pascal's triangle. Include a section on the relationship between Pascal's triangle and coin-tossing experiments.