Essential Ideas

SUMS

1. Find each sum.

a. $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$

b. $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4$

c. $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5$

d. $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots + \left(\frac{1}{2}\right)^n$

2. This sum goes on for ever. (We call it an *infinite series.*) Use the pattern you found in problem 1 to estimate the sum of this infinite series.

$$\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots$$

3. Estimate the sums of these infinite series.

a. $\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \ldots$

b. $\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \ldots$

c. $\frac{1}{k} + \left(\frac{1}{k}\right)^2 + \left(\frac{1}{k}\right)^3 + \ldots$

(Assume that k is a positive integer.)

GEOMETRIC SEQUENCES

4. Some of the following sequences are geometric; find their common ratio. Some are arithmetic; find their common difference.

a. 2/3, $(2/3)^2$, $(2/3)^3$, $(2/3)^4$, ...

b. 1/3, 4/3, 7/3, 10/3, ...

c. 10, 10/8, 10/64, 10/512, ...

d. 10, 80, 640, 5120, ...

e. 1/3, 8/3, 64/3, 512/3, ...

5. Find the sum of the first 50 terms for the sequences in problems 4a and e.

6. Two of the sequences in problem 4 are such that if you add the entire infinite sequence, the sum converges to a finite number.

a. Explain how you can tell which sequences they are.

b. Find the sum they each converge to.

INHERITANCE

The brothers Able and Earl inherited from their father an acre of land, which they divided equally. Each brother willed his land to his family. Able's family was large, and Earl's was small. Able's family needed more land, so they bought 40% of the land belonging to Earl's family. In the next generation, Able's family again bought 40% of Earl's family land. This continued for several generations.

7. Copy and and extend this table to show the amount of land owned by each family up to the eighth generation.

Generation	Able's land	Earl's land
1	0.5	0.5
2	0.7	0.3

8. Study the data. At this rate, will Able's family ever own the whole acre? Explain.

DECIMALS AND FRACTIONS

9. Write as a fraction.

a. $0.\overline{21}$

b. $0.3\overline{21}$

c. $0.\overline{321}$

10. Find whole numbers p and q such that:

a. $0.45 < p/q < 0.46$

b. $0.\bar{4} < p/q < 0.45$

PRIME FACTORIZATION

11. Explain why the square of an even number must be a multiple of four.

12. Explain why the square of an odd number must be odd.

13. Explain why the double of an odd number is an even number, but not a multiple of four.

LATTICE POINTS

Imagine that you are standing at the origin, and that you cannot see lattice points that are hidden behind other lattice points. For example, you cannot see (2, 2) because (1, 1) is in the way. Let us call (1, 1) *visible* and (2, 2) *hidden*.

14. List three visible lattice points and three hidden ones. Explain.

15. By looking at its coordinates, how can you tell whether a lattice point is visible?

16. Give the equation of a line that includes no lattice points except the origin.

17. Give the equation of a line that includes no lattice points at all.

18. Which line on an 11-by-11 geoboard contains the greatest number of visible lattice points?

GAMES AND PROBABILITY

19. If you choose a letter at random from the alphabet, what's the probability that it's a vowel?

20. If you choose a month at random, what's the probability that its name

a. begins with J?

b. contains an R?

21. Assume that you draw one card from an ordinary deck of 52 playing cards. What's the probability that you draw

a. a 7?

b. a heart?

c. a 7 or a heart?

d. a 7 of hearts?

22. Which game, if either, is fair? Explain.

a. Roll a pair of dice and multiply the numbers on the uppermost faces. If the product is 18 or greater, Player A wins. If the product is less than 18, Player B wins.

b. Toss three coins. If the number of heads is even, Player A wins. If it is odd, Player B wins.

c. Repeat part (b) for six coins.

UNIT CONVERSION

23. Given that 1 pound is approximately 454 grams, 1 kilogram is approximately how many pounds?

24. Find conversion factors for converting the following measurements. (Note: Even though these problems look different, you can use the technique you learned in Lesson 8. Remember that $in.^2$ means in. · in.)

a. $in.^2$ to ft^2 b. ft^2 to $in.^2$

c. $in.^3$ to cm^3 d. cm^3 to $in.^3$

25. The density of water is approximately 1 $gram/cm^3$. What is it in $pounds/ft^3$?