LESSON 13.3

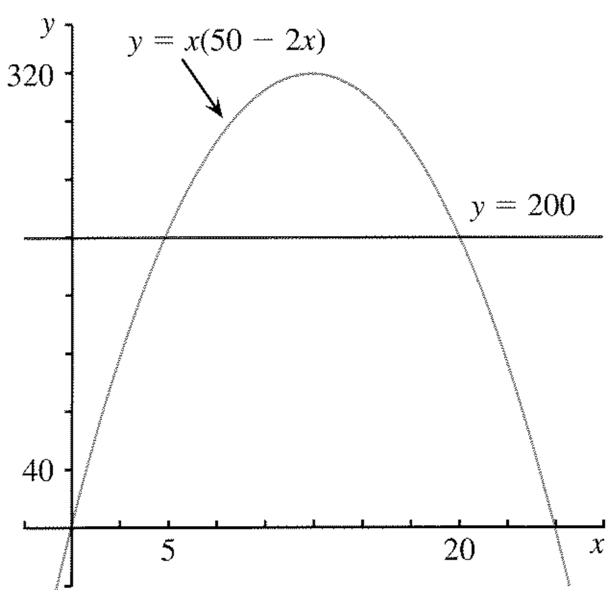
The Zero Product Property

You will need:

- graph paper
- or a graphing calculator (optional)

1. Given that you have 50 feet of fencing and that you can use the wall of the garage for the fourth side of your pen, what dimensions should you choose to make a rectangular pen having area 200 square feet? Solve by trial and error or by graphing. (There is more than one solution.)

This problem can be solved by writing the equation $x(50 - 2x) = 200$, where x is the distance from the wall to the side opposite it. One way of doing it is to find the intersection of the graphs of $y = x(50 - 2x)$ and $y = 200$.



2. Use this method to find the dimensions for the following areas:

a. 300   b. 250

Another method of solving this kind of problem is to write a quadratic equation and factor it, as explained in the following sections.

ZERO PRODUCTS

3. If $ab = 0$, which of the following is impossible? Explain.

a. $a \neq 0$ and $b \neq 0$

b. $a \neq 0$ and $b = 0$

c. $a = 0$ and $b \neq 0$

d. $a = 0$ and $b = 0$

Zero Product Property: When the product of two quantities is zero, one or the other quantity must be zero.

An equation like $(x + 6)(2x - 1) = 0$ can be solved using the zero product property. Since the product in the equation is zero, you can write these two equations.

$$x + 6 = 0 \quad \text{or} \quad 2x - 1 = 0$$

4. You know how to solve these equations. Write the solutions.

5. There are two solutions to the equation $(x + 6)(2x - 1) = 0$. What are they?

Solve these equations.

6. $(3x + 1)x = 0$

7. $(2x + 3)(5 - x) = 0$

8. $(2x - 2)(3x - 1) = 0$

SOLVING QUADRATIC EQUATIONS

Some quadratic equations can be solved using the zero product property.

Example: Find the values of x for which

$$x^2 + 6x = -5.$$

First rewrite the equation so you can apply the zero product property.

$$x^2 + 6x + 5 = 0$$

In factored form, this is written:

$$(x + 5)(x + 1) = 0.$$

Since the product is 0, at least one of the factors must be 0. So $x + 5 = 0$ or $x + 1 = 0$.

9. What are the two solutions of the equation $(x + 5)(x + 1) = 0$?

Example:

Find the values of x for which $6x^2 = 12x$.

First rewrite the equation so that you can apply the zero product property.

$$6x^2 - 12x = 0$$

In factored form, this is written:

$$6x(x - 2) = 0.$$

10. What are the two solutions to the equation $6x(x - 2) = 0$?

11. Factor and use the zero product property to solve these quadratic equations.

a. $x^2 - x = 2$

b. $2L^2 - L = 3$

c. $W^2 + 10W + 16 = 0$

d. $3M^2 + 30M + 48 = 0$

To solve problem 2a, write the equation:

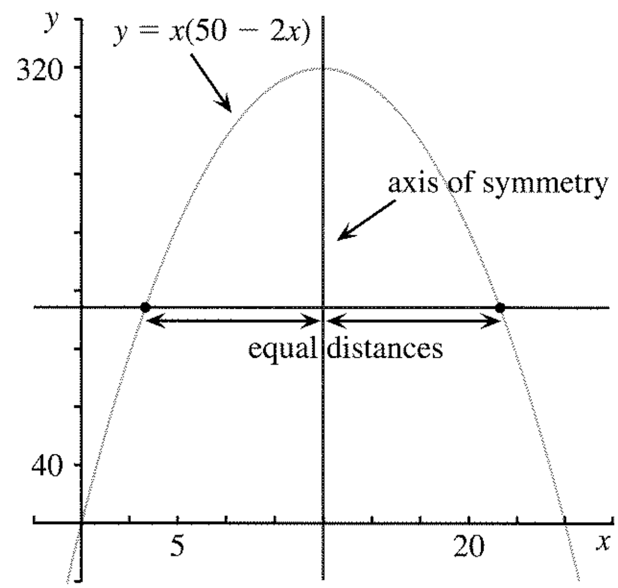
$$x(50 - 2x) = 300$$
$$-2x^2 + 50x - 300 = 0$$
$$2x^2 - 50x + 300 = 0$$
$$x^2 - 25x + 150 = 0$$

12. Explain the four steps.

13. Factor the final equation and use the zero product property to solve it.

Unfortunately, most quadratic equations cannot easily be solved by factoring. In the next chapter you will learn a way that always works to solve quadratic equations.

SYMMETRY



The vertical line through the vertex of a parabola is called its *axis of symmetry*.

14. How far is each x-intercept from the axis of symmetry in the preceding graph?

The x-intercepts are *equidistant* from the axis of symmetry. (They are at an equal distance from it.) As you can see in the figure, this is also true of any pair of points of the parabola that lie on the same horizontal line as each other.

15. Refer to the graph for problem 1.

a. Show that the two solutions to problem 1 are equidistant from the axis of symmetry.

b. Is this also true of the two solutions to problem 2a? What about problem 2b? Show your work.

VERTEX AND INTERCEPTS

In an equation like $y = 2(x + 3)(x - 4)$, you can quickly find the intercepts and the vertex.

16. What is the value of x at the y-intercept? Substitute this value for x in the equation and find the y-intercept.

17. What is the value of y at the x-intercepts? Substitute this value for y in the equation and find the x-intercepts with the help of the zero product property.

18. If you know the x-intercepts, how can you find the x-coordinate of the vertex? Find it.

19. If you know the x-coordinate of the vertex, how can you find its y-coordinate? Find it.

20. Find the intercepts and vertex for:

a. $y = 0.5(x - 0.4)(x - 1)$

b. $y = 2(x + 3)(x + 4)$

21. Explain how you would find the intercepts and vertex for a function of the form

$$y = a(x - p)(x - q).$$

22. Find the equation and the vertex for a parabola having the following intercepts:

a. (3, 0), (6, 0), (0, 36)

b. (3, 0), (6, 0), (0, 9)

c. (-3, 0), (-6, 0), (0, -9)

d. (-3, 0), (6, 0), (0, 6)

23. The vertex and one of the two x-intercepts of parabolas are given. Find the equation and the y-intercept.

a. vertex: (2, -2); x-intercept: (1, 0)

b. vertex: (1, -12); x-intercept: (-1, 0)

c. vertex: (3, 4.5); x-intercept: (6, 0)

DISCOVERY *TWO DEFINITIONS*

Definition: The absolute value of a number is the distance between that number and zero.

Browsing through Ginger's calculus book, Mary and Martin noticed this definition:

Definition: $|x| = \begin{cases} x \text{ for } x > 0 \\ -x \text{ for } x < 0 \end{cases}$

"That $-x$ must be a misprint," Mary commented. "Absolute value can't be negative."

24. Report Write a letter to Mary explaining everything you know about absolute value. Restate the two definitions presented above in your own words. Using examples, explain why they are equivalent, and why Mary was wrong about the misprint.