LESSON 14.1

Rectangle Ratios

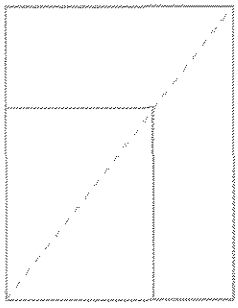
You will need:

scissors

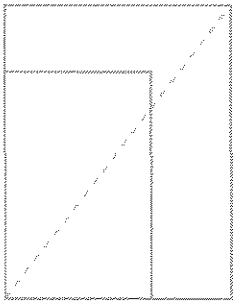
graph paper

HALF RECTANGLES

1. Take two identical rectangular pieces of paper. Fold one in half. Place it on top of the other piece. Is the folded half-rectangle similar to the original rectangle? Check with the diagonal test.



similar



not similar

2. **Exploration** Make a paper rectangle, such that the rectangle you get by folding it in half is similar to the original rectangle. What are the dimensions of your rectangle? (Hint: Remember that if two rectangles are similar, their length-to-width ratio must be the same. You may use trial and error on your calculators for different sizes.)

3. a. Sketch a 16-unit-by-12-unit rectangle on graph paper. What is the length-to-width ratio?
   b. Divide the rectangle in half to get one having length 12 and width 8. (The width of the original rectangle becomes the length of the new rectangle.) What is the length-to-width ratio?
   c. Continue to divide the rectangle in half, calculating the ratio of length to width. Make a table like this one to record your data.

Length	Width	*l*/*w* Ratio
16	12	—
12	8	—

4. Describe any patterns you notice in your table.

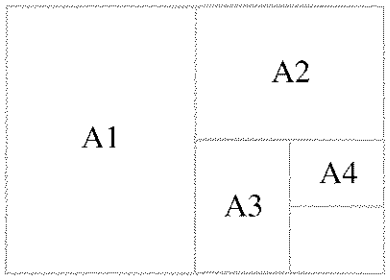
5. a. Repeat problem 2 for three more rectangles. Keep a careful record of your data in tables. Look for patterns.
   b. Find some rectangles for which the length-to-width ratios do not change when you cut them in half.

6. A rectangular sheet of paper is 1 foot wide and x feet long. It is cut into two rectangles, each of which is $(1/2)x$ feet wide and 1 foot long.
   a. Illustrate this in a diagram.
   b. What is the length-to-width ratio in the original rectangle?
   c. What is the length-to-width ratio in each of the two new rectangles?
   d. If the rectangles are similar, we can write an equation setting the original ratio equal to the new ratio. Do this, and find the value of x. Show your calculations.

7. **Report** Summarize your findings from problems 1 through 6. Include sketches and examples. Describe any patterns you noticed. For the rectangles you found in problem 5b, what was the common ratio? What was the common ratio for the rectangle you found in problem 2?

THE INTERNATIONAL PAPER STANDARD

In 1930 an international standard was established for paper sizes, called the *A-series*. The basic size is A0, which is one square meter in area. If you fold it in half, you get paper of size A1. You can fold A1 in half to get A2, fold A2 in half to get A3, etc. The dimensions of A0 were chosen so that *all paper sizes in the series are similar to each other and to A0.*

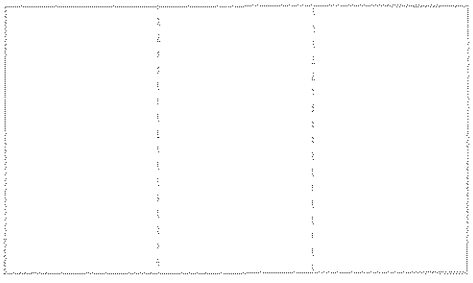


8. Find the dimensions, to the nearest millimeter, of A0 and A1.

DYNAMIC RECTANGLES

The special rectangles you discovered in the previous section each have the property that half of the rectangle is similar to the whole. They are examples of a group of rectangles, called *dynamic rectangles*, that are very useful to artists and designers. Dynamic rectangles have the property that when you cut them into a certain number of equal parts, each of the parts is similar to the whole.

The rectangle below is divided into three parts, each one of which is similar to the original rectangle.



We can express this similarity by writing two equal ratios.

$$\frac{L}{W} = \frac{W}{\frac{1}{3}L}$$

Multiplying both sides of the equation by W:

$$W\left(\frac{L}{W}\right) = W\left(\frac{W}{\frac{1}{3}L}\right)$$

$$L = \frac{W^2}{\frac{1}{3}L}$$

and then by $\frac{1}{3}L$, we get the equation:

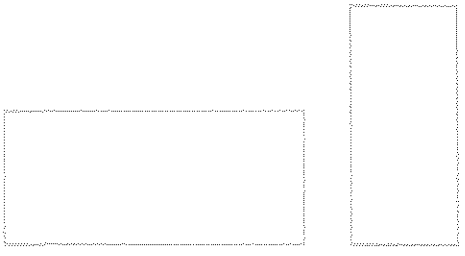
$$\frac{1}{3}L^2 = W^2$$

9. Show how to find L, the length of the original rectangle, if the width is the following:

a. 1 b. 2 c. W

10. What is the ratio of length to width in each of the rectangles in problem 9?

Dynamic rectangles are named for their ratio of length to width. These two rectangles are both called $\sqrt{5}$ rectangles because the ratio of length to width in each of them is $\sqrt{5}$.



11. Into how many equal parts would you divide a $\sqrt{5}$ rectangle in order to make each of the parts similar to the original rectangle? Explain how you figured this out, showing your work.

12. A rectangle is divided into seven parts, each of which is similar to the original rectangle.

a. Give possible dimensions (length and width) for the rectangle.

b. Give another set of possible dimensions.

c. What is the ratio of length to width?

13. A rectangle having width one unit is divided into n equal parts, each of which is similar to the original rectangle.

a. To find the length x of the original rectangle, Tara wrote:

$$\frac{x}{1} = \frac{1}{\frac{1}{n}x}$$

Explain why Tara wrote this proportion.

b. Solve this equation for x.

c. Summarize your results in words.

14. **Research** Many artists and designers use mathematics. Do some research to find out why dynamic rectangles are so useful in art and design. Then make your own design based on dynamic rectangles.

DISCOVERY ***INTERESTING NUMBERS***

15. Find a number that is one more than its reciprocal.

16. Find a number that is one less than its square.

REVIEW ***NUMBERS AND THEIR RECIPROCALS***

If possible, find or estimate the number described. Explain how you found it. (If there is more than one number that fits the description, try to find as many as possible.)

17. The number equals its reciprocal.

18. The number is four more than its reciprocal.

19. The number is one more than twice its reciprocal.

20. The number does not have a reciprocal.