Knowing more about quadratic functions and their graphs will help you understand and solve quadratic equations. In particular, it is useful to know how to find the vertex and the x-intercepts of quadratic functions in the following two forms:

- **Intercept form**: \( y = a(x - p)(x - q) \)
- **Standard form**: \( y = ax^2 + bx + c \)

### Different Shapes

1. The figure shows several parabolas whose x-intercepts, y-intercept, and vertex are all (0, 0). Match each one with an equation:
   - \( y = x^2 \)
   - \( y = 0.5x^2 \)
   - \( y = 2x^2 \)
   - \( y = -x^2 \)
   - \( y = -0.5x^2 \)
   - \( y = -2x^2 \)

2. What is the value of \( a \) for the parabolas on the following figure?

### Summary

As you learned in Chapter 13, when the equation is in intercept form, you can find the vertex from the x-intercepts, which are easy to locate.

7. Try to answer the following questions about the graph of \( y = 2(x - 3)(x + 4) \) without graphing.
   - a. What are the x- and y-intercepts?
   - b. What are the coordinates of the vertex?
8. **Generalization**
   a. What are the x- and y-intercepts of \( y = a(x-p)(x-q) \)? Explain.
   b. Explain in words how to find the vertex if you know the intercepts.

9. □ The figure shows the graphs of several parabolas. Write an equation for each one. (Hint: To find \( a \), use either the y-intercept or the vertex and algebra or trial and error.)

![Graphs of Parabolas](image)

10. For each equation, tell whether its graph is a smile or a frown parabola, without graphing. Explain your reasoning.
   a. \( y = 9(x-8)(x-7) \)
   b. \( y = -9(x-8)(x-7) \)
   c. \( y = 9(8-x)(x-7) \)
   d. \( y = 9(8-x)(7-x) \)

11. □ If you know all the intercepts and the vertex of \( y = 3(x-p)(x-q) \), explain how you would find the intercepts and the vertex of \( y = -3(x-p)(x-q) \).

### STANDARD FORM

When the equation is in standard form, \( y = ax^2 + bx + c \), it is more difficult to find the location of the vertex. One particularly easy case, however, is the case where \( c = 0 \).

12. □ Explain why when \( c = 0 \), the parabola goes through the origin.

13. Find the vertex of \( y = 2x^2 + 8x \).
   (Hint: Factor to get into intercept form.)

![Graphs of Parabolas](image)

14. □ How are the two graphs related? Compare the axis of symmetry and the y-intercept.

15. □ How is the graph of \( y = 2x^2 + 8x - 3 \) related to them?

16. Find the equation of any other parabola whose vertex is directly above or below the vertex of \( y = 2x^2 + 8x \).

### FINDING \( H \) AND \( V \)

**Example**: Find the coordinates \((H, V)\) of the vertex of the graph of \( y = 3x^2 - 18x + 7 \).
- \( y = 3x^2 - 18x \) is the vertical translation for which \( V = 0 \). By factoring, we see it is equal to \( y = x(3x - 18) \).
- To find the x-intercepts of \( y = 3x^2 - 18x \), we set \( y = 0 \). By the zero product property, one x-intercept is 0. To find the other, we solve the equation \( 3x - 18 = 0 \), and get \( x = 6 \).
- Since the x-intercepts are 0 and 6, and the axis of symmetry for both parabolas is halfway between, it must be 3. So \( H = 3 \).
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• Substitute 3 into the original equation to see that the $y$-coordinate of the vertex is:
  $$V = 3(3)^2 - 18(3) + 7 = -20.$$ 
  So the coordinates of the vertex for the original parabola are $(3, -20)$.

17. For each equation, find $H$ and $V$. It may help to sketch the vertical translation of the parabola for which $V = 0$.
  
  a. $y = x^2 + 6x + 5$
  b. $y = 2x^2 + 6x + 5$
  c. $y = 3x^2 - 6x + 5$
  d. $y = 6x^2 - 6x + 5$

**Generalizations**

18. What is the equation of a parabola through the origin that is a vertical translation of $y = ax^2 + bx + c$?

19. Show how to find the axis of symmetry of:
   
   a. $y = ax^2 + bx$
   b. $y = ax^2 - bx$.

20. Explain why the $x$-coordinate of the vertex of the parabola having equation $y = ax^2 + bx + c$ is
   $$H = -\frac{b}{2a}.$$ 

**SAME SHAPE**

The parameter $a$ determines the shape of the parabola. The graphs of all equations in standard form that share the same value for $a$ are translations of the graph of $y = ax^2$.

For example, the two parabolas in the figure have equations with $a = 0.25$. Therefore they have the same shape, as the following exercise shows.

21. 
   
   a. Show algebraically that starting at the vertex, and moving 4 across and 4 up, lands you on a point that satisfies the equation in both cases.
   b. If you move 2 across from the vertex, show that you move up the same amount to get to the parabola in both cases.