Essential Ideas

WINDOWS AND PANES

1. The A.B. Glare Window Store sells a two-pane window, especially designed so that the panes have the same dimensions as each other, and the whole window has the same proportions as each pane. If the horizontal dimension of the window is 36 inches, what is the vertical dimension, to the nearest inch? Make a sketch and show your work.

2. The A.B. Glare Window Store sells two models of two-pane windows, such that one pane is square and the other is rectangular. The rectangular pane has the same proportions as the whole window. Both models have a horizontal dimension of 36 inches. Make a sketch and show your work as you answer the following question: What are the dimensions of the rectangular pane, if its longer dimension is a. horizontal? b. vertical?

ALGEBRAIC FRACTIONS

3. Dwight was simplifying \( \frac{x+2}{x} \). He said, "I can't get rid of the x's in the numerator and denominator." He wrote \( \frac{x+2}{x} = 2. \) Did Dwight correctly simplify \( \frac{x+2}{x} \)? Is his statement always, sometimes, or never true?

If possible, simplify the fractions.

4. \( \frac{xy + y}{y} \)
5. \( \frac{3x + 3y}{x^2 - y^2} \)
6. \( \frac{3a + 3b}{4a + 4b} \)
7. \( \frac{6}{6x - 6} \)
8. \( \frac{x^2 + 5x}{x^2 + 4x} \)
9. \( \frac{2x + 2y}{3x + 3y} \)

ALWAYS, SOMETIMES, NEVER

Tell whether each expression is always, sometimes, or never true.

10. \( \frac{3x + 5}{3x} = 5 \)
11. \( \frac{3x + 3y}{x + y} = 6 \)
12. \( \frac{3x + 3y}{x + y} = 3 \)
13. \( \frac{3x + y}{y} = 3x \)

EQUIVALENT FRACTIONS

14. Write a fraction having a denominator of 6y that is equivalent to:
   a. \( \frac{1}{6} \)
   b. \( x \)

15. Write a fraction having a denominator of y that is equivalent to:
   a. \( 6x \)
   b. \( 6xy \)

16. a. Write a fraction equivalent to \( \frac{3}{x} \) having \( xy \) as a denominator.
   b. Write a fraction equivalent to \( \frac{5}{y} \) having \( xy \) as a denominator.
   c. Add \( \frac{3}{x} \) and \( \frac{5}{y} \). (Hint: To add, you need a common denominator.)

17. a. Write a fraction that is equivalent to \( x \) having \( x \) as a denominator.
   b. Add \( x + \frac{1}{x} \). (Hint: Find a common denominator.)

Put on the same denominator.

18. \( \frac{x^2 + b}{a} + \frac{c}{a} \)
19. \( \frac{-b^2}{4a^2} + \frac{c}{a} \)

DIVISION BY ZERO?

On a test Joel solved the quadratic equation \( 6x^2 = 12x \) using this method:

Divide both sides by \( x \): \( \frac{6x^2}{x} = \frac{12x}{x} \)
Simplify fractions: \( 6x = 12 \)
Divide both side by 6: \( \frac{6x}{6} = \frac{12}{6} \)
The answer is \( x = 2 \).
Joel’s teacher, Mr. Letter, wrote this on his paper:

There are two solutions to this equation. You missed one of them because you divided by 0.

Joel was puzzled. “I divided by $x$, and then by 6” he thought. “I never divided by 0.”

20. Can you explain what Mr. Letter meant? Can you solve the equation correctly?

**Mystery Parabolas**

Make a rough sketch showing two parabolas having the features described. Some of your parabolas should be frowns and others smiles; some should be more open, some less. Label each parabola with:
- its equation;
- its axis of symmetry;
- its $x$-intercepts (exact values);
- its vertex.

21. The parabola has $x$-intercepts at 2 and −4.

22. The parabola has vertex (3, −5).

23. The parabola has an $x$-intercept at $\sqrt{5}$ and is symmetric with respect to the $y$-axis.

24. The parabola has an $x$-intercept at $1−\sqrt{6}$ and has the line $x = 1$ as its axis of symmetry.

25. The parabola has an axis of symmetry at $x = 5$ and $y$-intercept 3.

**Parabola Features**

26. Give the vertex, $x$-, and $y$-intercepts of:
   a. $y = 2(x + 3)^2 − 9$
   b. $y = 4(x − 5)(x + 1)$
   c. $y = 6x^2 − 7x − 8$

27. How many $x$-intercepts?
   a. $y = -2(x + 3)^2 − 9$
   b. $y = -4(x − 2)$
   c. $y = 6x^2 + 7x + 8$

**From Fractions to Quadratics**

Rewrite each equation as an equivalent quadratic equation. Then solve the equation. Show your work.

28. $w + 9 = \frac{10}{w}$
29. $L + 3 = 2 + \frac{6}{L}$
30. $L − 4 = \frac{32}{L}$
31. $\frac{1}{x} = x − 1$

Solve these equations. They have zero, one, or two solutions.

32. $\frac{4}{x} + x = −4$
33. $\frac{1}{x} + \frac{2}{x} = \frac{3}{x}$
34. $1 = \frac{1}{x} + \frac{1}{x^2}$

**Write an Equation**

35. Write a quadratic equation that has the following solutions:
   a. 4 and −2
   b. $\sqrt{5}$ and $-\sqrt{5}$
   c. $1 + \sqrt{5}$ and $1 − \sqrt{5}$

36. Write a quadratic equation that has the solution −6.

37. Write a quadratic equation that has no real number solutions.