Maximizing Area

1. You want to make a rectangular pen for Stripe, your pet zebra. You want to make sure she has as much space as possible inside the pen. You have 50 feet of fencing available. If you use all of it to make the pen, what is the biggest area possible?

2. Make a graph of area as a function of length. Connect the points with a smooth curve. What kind of curve is it?

3. Interpret the graph:
   a. Label the highest point on your graph with its coordinates. Interpret these two numbers in terms of this problem.
   b. Where does the graph cross the x-axis? What do these numbers mean?
   c. If you increase the length by 1 foot, does the area increase or decrease? Does it change by the same amount each time? Explain.

4. If the perimeter of a rectangular pen is 28:
   a. Write an algebraic expression for its area in terms of the length L.
   b. If you had 28 feet of fencing and wanted to make the largest possible rectangular pen, what would its length, width, and area be? Explain.

5. **Generalization.** Say the perimeter of a rectangle is P and its length is L.
   a. Express the width in terms of P and L.
   b. Express the area in terms of P and L.
   c. Express the maximum area in terms of P only.

Assume that you have 50 feet of fencing to build a rectangular pen. You plan to use the garage wall as one side of the pen, which means you only need to use your fencing for three of the four sides. You will make the partitions inside the pen, at a right angle to the wall.

6. Make a rough sketch of what this pen might look like,
   a. with no internal partitions
   b. divided into two sections

7. With no partitions, is it possible to get a square pen? If so, what are its dimensions?

8. With one partition is it possible to get two square sections? If so, what are their dimensions?

Call the side of the pen parallel to the wall the length, and the distance between the wall and the side opposite the wall x.

9. If you have to use part of the 50 feet of fencing for a partition to divide the pen into two equal parts, what is the largest total area you can get for the enclosure? Explain how you got your answer, including a graph if necessary.

10. Solve the above problem if you want to divide the pen into 3 equal parts.

11. Solve the above problem if you want to divide the pen into n equal parts.

12. Look at your solutions for problems 9, 10, and 11. In each case, look at the shapes of the subdivisions of the maximum-area pen. Are they always squares? Are they ever squares? Does the answer to this depend on the value of n? Explain.

13. Look at your solutions for problems 9, 10, and 11. In each case, look at how much of the fencing was used to construct the side parallel to the garage for the pen of maximum area. What fraction of the fencing was used to construct the side parallel to the garage? Does the answer depend on the value of n? Explain.