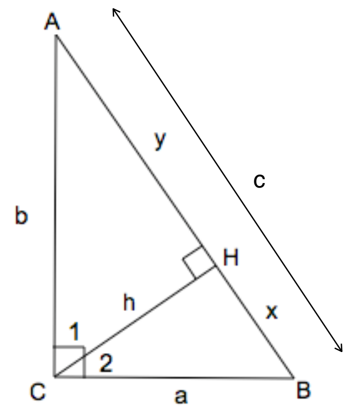
The Three Triangles

Here is one way to prove the Pythagorean Theorem using your newfound knowledge of scaling and similarity.

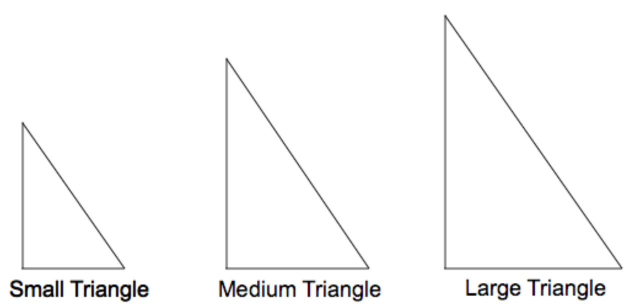
The figure below shows three triangles. The points are indicated by capital letters, and the side lengths by lower case letters.



1. Copy the original diagram and be sure that your diagram is clearly labeled.

2. Write as many relationships as you can involving the figure's acute angles ($\angle A$, $\angle B$, $\angle 1$, $\angle 2$). Find at least six different equations and explain them (example: $\angle 1 + \angle 2 = 90^\circ$) .

3. There are three triangles in this diagram. Draw them separately and line up the corresponding parts. Be sure to label the sides and angles of all three triangles using the appropriate upper and lower case letters. Your triangles should look something like this:



4. Explain why the triangles are similar to each other.

5. There are three pairs of similar triangles. For each pair, write that their sides are proportional by writing the ratios in the format:

$$\frac{SHORT\ LEG}{short\ leg} = \frac{LONG\ LEG}{long\ leg} = \frac{HYPOTENUSE}{hypotenuse}$$

Using the proportions and the Pythagorean Theorem

6. If $a = 4$ and $b = 5$, find all of the other lengths (h, x, y, and c).

7. If $x = 4$ and $y = 5$, find all of the other lengths.

Finding Formulas

Example: I would like to find a formula involving h^2. I look in the proportions that I found in #5 and I find a pair of equal fractions where h appears twice:

$$\frac{x}{h} = \frac{h}{y}$$

I can solve for h^2 by cross-multiplying and I find the following relationship: $h^2 = xy$.

8. Follow the same kind of logic as in the example to complete these formulas:
   $a^2 =$ ______
   $b^2 =$ ______
   $hc =$ ______

9. What is the area of the big triangle in terms of
   (a) a and b?
   (b) c and h?
   (c) How is this related to #8? Explain.

10. A proof of the Pythagorean Theorem:
    Use the relationships for a^2 and b^2 that you found in #8.
    Write an equation for side c that does not include a and b.
    Do a little algebra.

Super Triples

11. {8, 15, 17} is a Pythagorean Triple.
    (a) What does this mean?
    (b) Is {7, 24, 25} a Pythagorean triple? Why or why not?
    (c) Name three other Pythagorean triples. Check the triples that your neighbor finds.

Definition: A *super triple* is a Pythagorean triple such that not only a, b, and c, but also x, y, and h are whole numbers (see your original figure).

12. **Challenge**: Find some super triples.