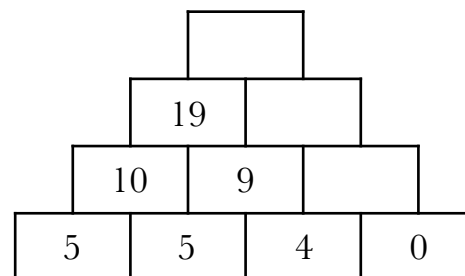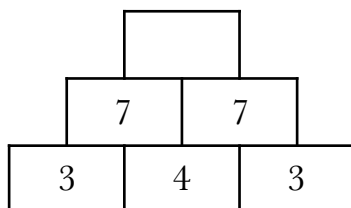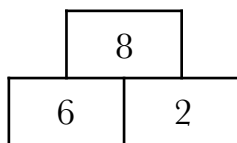# Number Pyramids

In a number pyramid, each number is the sum of the two numbers directly beneath it. Let's call that the *pyramid rule.* Here are some examples.
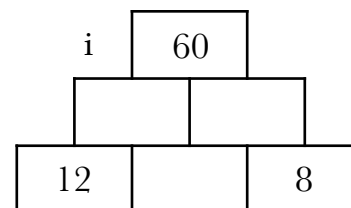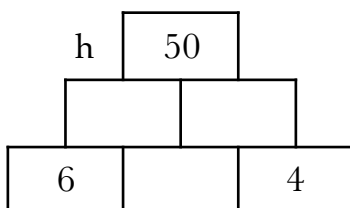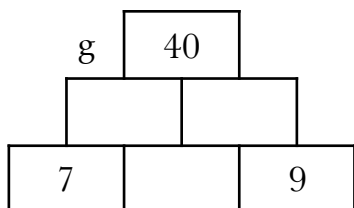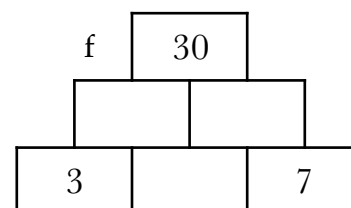
	8	
6	2	

Pyramid 2:
```
        [ ]
     7      7
   3     4     3
```

Pyramid 3:
```
              [ ]
         19        [ ]
      10     9        [ ]
    5     5     4     0
```

1. Fill out the empty cells in the examples.

If you did this correctly, the top number in the last pyramid should be 32.

Pyramid Puzzles

The given numbers are called *seeds.* You are given three seeds in each pyramid:

a.
```
      10
    4    [ ]
  1   [ ]   [ ]
```

b.
```
      10
   [ ]   [ ]
  2   [ ]   4
```

c.
```
      20
   [ ]   [ ]
  3   [ ]   5
```

d.
```
      10
    4    [ ]
 [ ]  [ ]   5
```

e.
```
      20
   [ ]   [ ]
  2   [ ]   4
```

f.
```
      30
   [ ]   [ ]
  3   [ ]   7
```
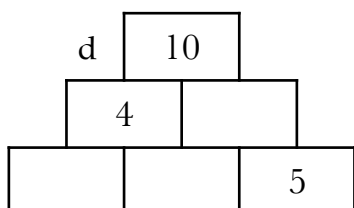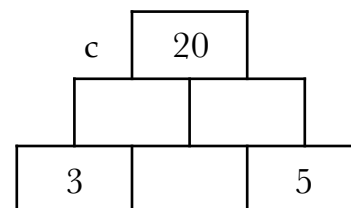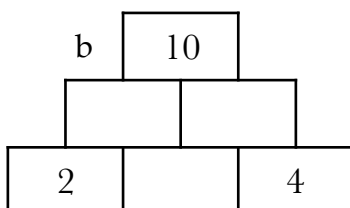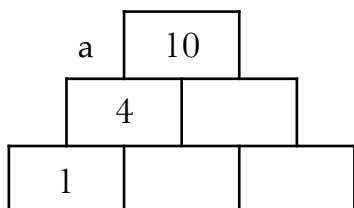
g.
```
      40
   [ ]   [ ]
  7   [ ]   9
```

h.
```
      50
   [ ]   [ ]
  6   [ ]   4
```

i.
```
      60
   [ ]   [ ]
 12   [ ]   8
```
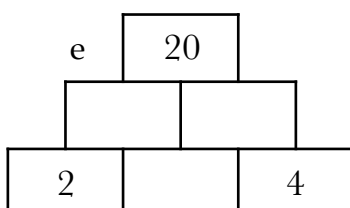
2. Fill out the empty cells in the pyramids above. (You don't have to do them in order.)

3. **Challenge:** Knowing the three corner numbers, it is possible to predict the number that will go in the middle of the bottom row without any trial and error. Explain how. Check that your idea works in all the pyramids.

Henri Picciotto, www.mathed.page

More Pyramid Puzzles

a — row 2: _, 20; bottom row: 1, 3, _, 5

c — top: 35; row 3: _, 7; bottom: 3, 5, _, _

b — row 2: 40, _; row 3: _, 9; bottom: 5, _, 7, _

d — top: 50; row 2: _, 22; bottom: 4, _, 6, _

f — top: 50; row 3: _, 13; bottom: 8, _, 10, _

e — row 2: 50, _; row 3: _, 13; bottom: 8, _, 10, _

g — row 2: 20, 16; bottom: 1, _, _, 2

i — top: 70; row 3: _, 20, _; bottom: 7, _, _, 3

h — top: 60; row 2: 9, _, 11; bottom: 0, _, _, _

j — top: 50; row 2: 9, _; bottom: 0, _, _, 5

k — top: 70; row 3: _, 10, _; bottom: 30, _, _, 10
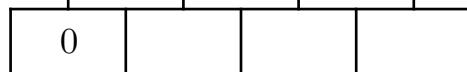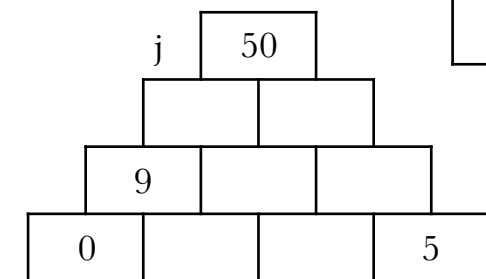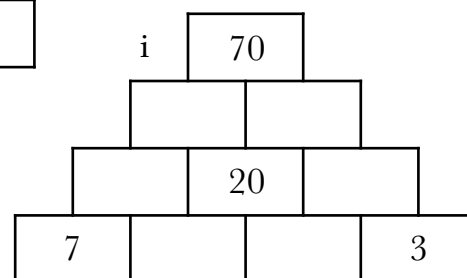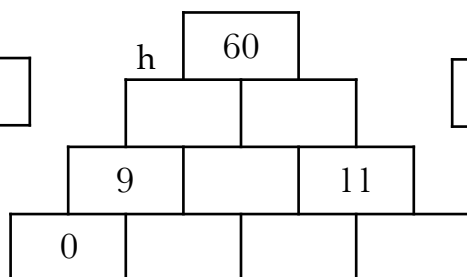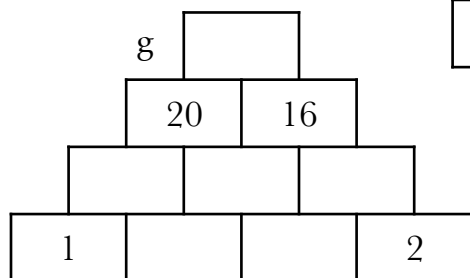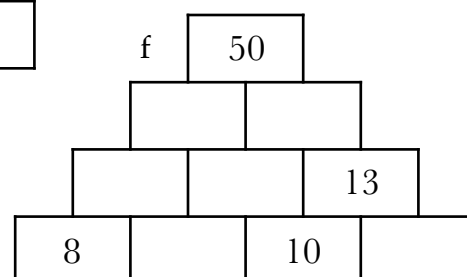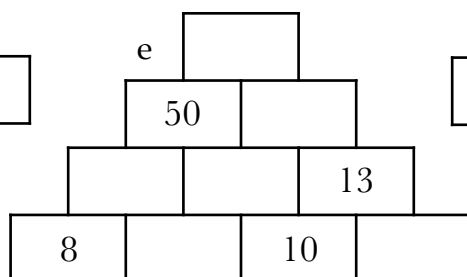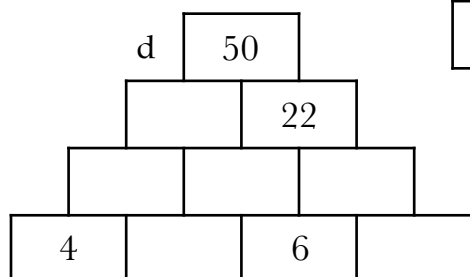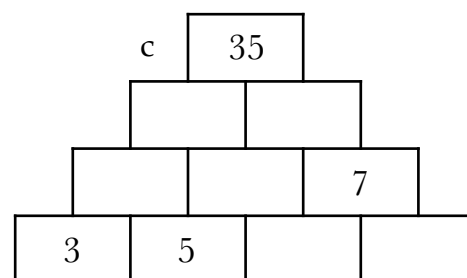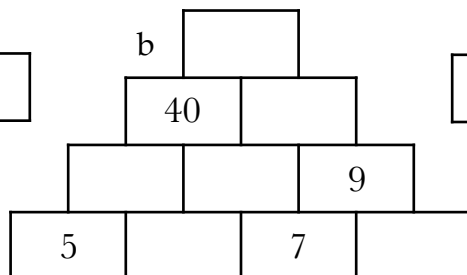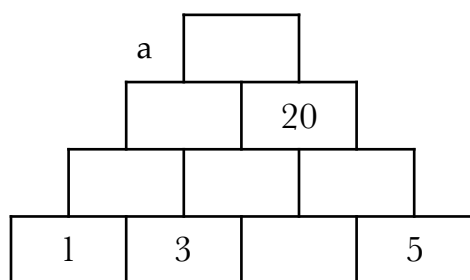
Fill out the empty cells in the pyramids above. (You don't have to do them in order.)

Hints:

◊ Start by solving smaller pyramids inside the given ones

◊ Use blank pyramids to experiment: enter the given numbers, plus a test number in the bottom row. Keep track of what you get at the top. Use that information to choose another test number. Keep going until you succeed!

Stuck in the Middle

In a four-row pyramid, only one cell does not touch the outside. Let's call it the *middle cell* and the number in it the *middle number*. Knowing the three corner numbers, it is possible to predict the middle number without any trial and error. This is what we will explore on this page.
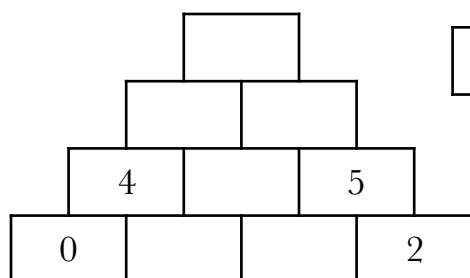
1. Study the pyramids on page 2, and try to discover how the middle number is related to the corner numbers. If you cannot figure it out, these additional examples may help.
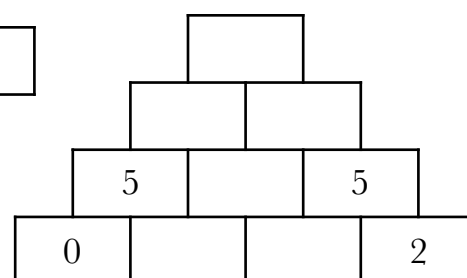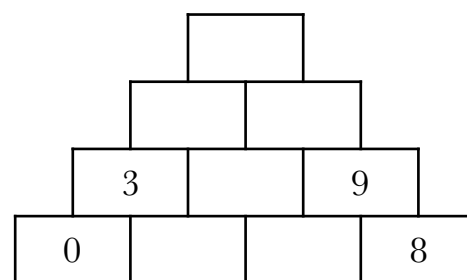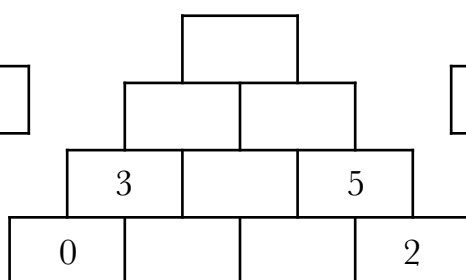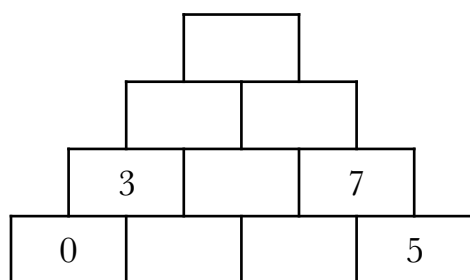
2. Fill in the blanks: In a four-row pyramid, the middle number is the top number, _____ the bottom corner numbers, divided _____. (Be sure to check that your idea works on the pyramids on page 2!)

3. One of the pyramid puzzles below cannot be solved, and the other can be solved in more than one way. Which is which? Explain.
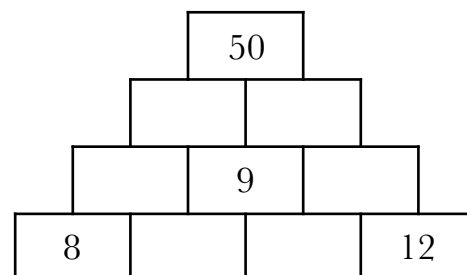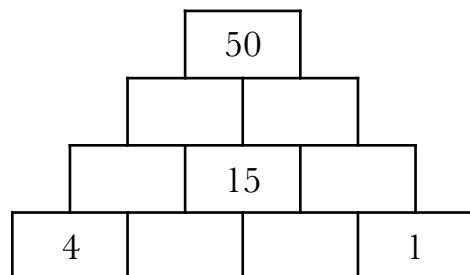
Making Pyramid Puzzles

We will explore making pyramid puzzles for students who are not familiar with the discoveries we made on previous pages. We will always start our pyramids with *non-negative whole number seeds*.

1. Most pyramid puzzles you have seen so far have just one solution. Among the above three, one is *impossible*: no matter how you start it, you can never fill it correctly. One has many different solutions — let's call it *undetermined*. And one has a unique solution, which is relatively easy to find, because it can be filled without much thinking. Let's call that one *automatic.* Which is which? How do you know each is which you claim it to be? How would you convince a friend?
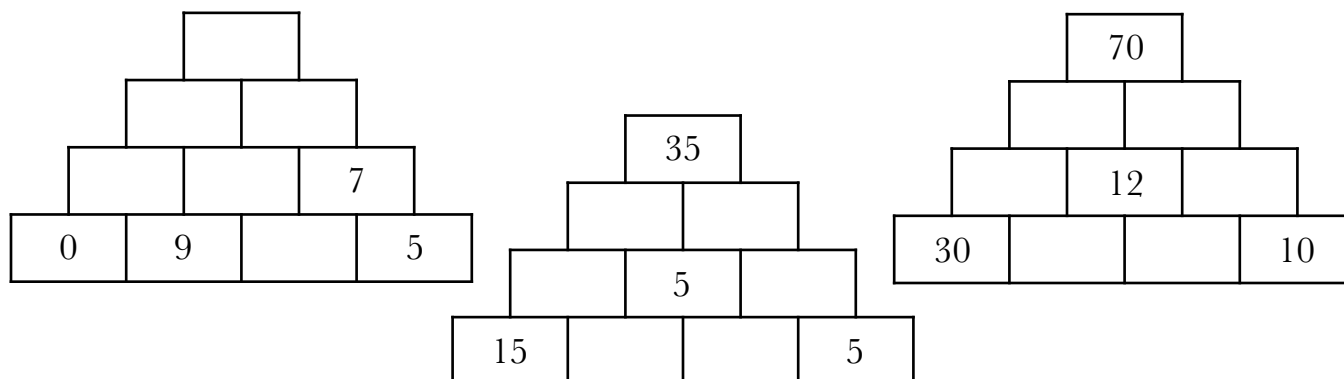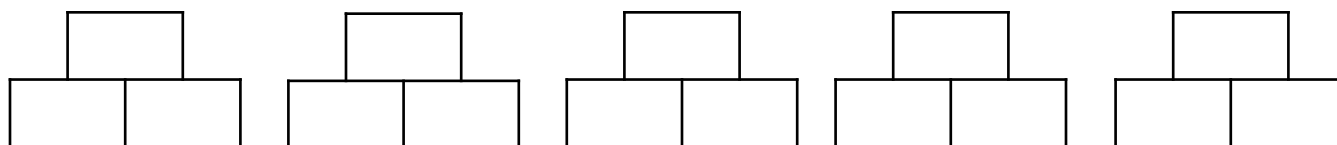
Two-Row Pyramids

2. What sort of puzzles can you make in a pyramid with just two rows? Answer the following questions.
    a. What happens if we start with just one seed?
    b. What happens if we start with two seeds? (Try different positions for the seeds.)
    c. In what cases do we end up with a negative number in the filled pyramid?

Three-Row Pyramids

What sort of puzzles can you make in a three-row pyramid?

3. Discuss what happens with three seeds, depending on where you place them.

4. Explore the situation if we start with two, three, or four seeds. Which is the best choice? Why?

5. In what cases do we end up with a negative number in the filled pyramid? How about a number that is not a whole number?
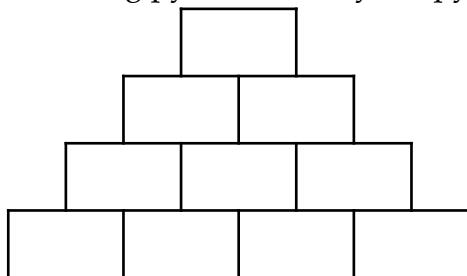
Making Four-Row Pyramid Puzzles

Using Existing Puzzles

1. Choose a puzzle from page 2, different from your neighbors', and make a new pyramid by multiplying all the seeds by the same number. (This is called *scaling*. The number you multiplied by is called the *scaling factor*.) Solve the resulting puzzle. How is the resulting pyramid related to the original?
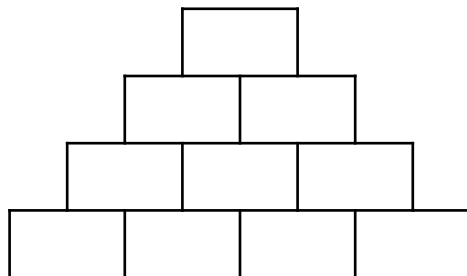
2. Choose two puzzles from page 2, and add the numbers in corresponding cells (all the cells, not just the seeds.) Does the resulting pyramid satisfy the pyramid rule?

3. Assume you want this pyramid to be the solution to a puzzle. Experiment to decide: how many seeds should you use? Where should you place them?

Challenge: Starting from Scratch

What sort of puzzles can you make in a four-row pyramid?

Hint: When working on the following problems, organize your experiments. For example, start with all seeds in the bottom row, then try to move one seed to another location. It is probably best to work with small numbers. Keep track of what happens!

4. Explore the situation if we start with two, three, four, or more seeds. Which is the best choice? Why?

5. Discuss what happens with four seeds, depending on where you place them. When is the puzzle easy? When is it impossible? When does it yield many solutions?

6. In what cases do we end up with a negative number in the filled pyramid? How about a number that is not a whole number?

Number Pyramids: Teacher's Guide

Print many copies of the **blank pyramids.pdf** document so that students can experiment as much as needed. Suggest that as they experiment, they use a different color for the seeds, or perhaps use a pen for the seeds, and pencil for the other numbers.

Depending on your class, your students may need some hints. Group work and/or whole-class discussion can help spread insights from a few students to their classmates.

Pages 1-3

These activities are intended for grades 4-8.

Page 1

Make sure everyone understands the pattern suggested in #3, as it will make page 2 much more manageable.

Page 2

Emphasize that the puzzles need not be solved in order. If students need help, you can remind them of the hints shown at the bottom of the page.

Six of the puzzles can be solved relatively easily using the embedded 2-row and 3-row pyramids. Two of the puzzles have many solutions, so it is not too difficult to find one. That leaves three that require trial and error.

As students work on those, they will notice that a larger seed yields a larger top number, which should help them adjust their guesses. Middle school students who have studied linear functions can be encouraged to make tables, with the seed and the top number. This should reveal a linear relationship.

Page 3

#2 constitutes a sledgehammer hint. If you think your students will not need it, you might ask for that result *before* handing out page 3, by having students analyze the pyramids on page 2.

Pages 4-5

These activities are intended for grades 6-8.

Page 4

#1 is largely based on the result from the previous page. Understanding the three types of situations is essential to the remaining activities.

Page 5

#1-3 are much more accessible than #4-6. #4-6 should be seen as a collective challenge for the whole class. It is not crucial to arrive at a complete and rigorous analysis of the problem: as they explore the situation, students should share any partial results.

Another version of #3 would be to add scaled versions of existing pyramids.

Acknowledgments