Escape from the Textbook!

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- Students are different
- Classes are different
- Teachers are different

One size does not fit all

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We need to be
flexible and eclectic...

3

... but much of the time,
we are prisoners of the textbook

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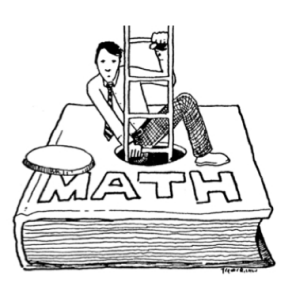
Escape from the Textbook!



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Do you escape...

- For a day?
- For a unit?
- For a whole course?

6

Do you escape...

- Alone?
- With colleagues?
- As a department?

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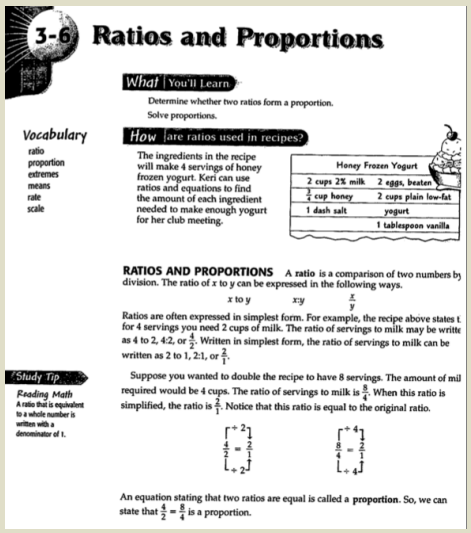

3-6 Ratios and Proportions

What You'll Learn

Determine whether two ratios form a proportion.
Solve proportions.

Vocabulary
ratio
proportion
extremes
means
rate
scale

How are ratios used in recipes?

The ingredients in the recipe will make 4 servings of honey frozen yogurt. Keri can use ratios and equations to find the amount of each ingredient needed to make enough yogurt for her club meeting.

Honey Frozen Yogurt	
2 cups 2% milk	2 eggs, beaten
$\frac{1}{2}$ cup honey	2 cups plain low-fat yogurt
1 dash salt	1 tablespoon vanilla

RATIOS AND PROPORTIONS A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.

x to y $\quad x:y \quad \frac{x}{y}$

Ratios are often expressed in simplest form. For example, the recipe above states t for 4 servings you need 2 cups of milk. The ratio of servings to milk may be writte as 4 to 2, 4:2, or $\frac{4}{2}$. Written in simplest form, the ratio of servings to milk can be written as 2 to 1, 2:1, or $\frac{2}{1}$.

Study Tip
Reading Math
A ratio that is equivalent to a whole number is written with a denominator of 1.

Suppose you wanted to double the recipe to have 8 servings. The amount of mil required would be 4 cups. The ratio of servings to milk is $\frac{8}{4}$. When this ratio is simplified, the ratio is $\frac{2}{1}$. Notice that this ratio is equal to the original ratio.

$$\frac{4}{2} \overset{\div 2}{=} \frac{2}{1} \qquad \frac{8}{4} \overset{\div 4}{=} \frac{2}{1}$$

An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{4}{2} = \frac{8}{4}$ is a proportion.

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Example 1 Determine Whether Ratios Form a Proportion

Determine whether the ratios $\frac{4}{5}$ and $\frac{24}{30}$ form a proportion.

$$\frac{4}{5} \overset{\left[\begin{smallmatrix}\div 1\\ \div 1\end{smallmatrix}\right]}{=} \frac{4}{5} \qquad \frac{24}{30} \overset{\left[\begin{smallmatrix}\div 6\\ \div 6\end{smallmatrix}\right]}{=} \frac{4}{5}$$

The ratios are equal. Therefore, they form a proportion.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

Example 2 Use Cross Products

Use cross products to determine whether each pair of ratios form a proportion.

a. $\frac{0.4}{0.8}, \frac{0.7}{1.4}$

$\frac{0.4}{0.8} \stackrel{?}{=} \frac{0.7}{1.4}$	Write the equation.
$0.4(1.4) \stackrel{?}{=} 0.8(0.7)$	Find the cross products.
$0.56 = 0.56$	Simplify.

The cross products are equal, so $\frac{0.4}{0.8} = \frac{0.7}{1.4}$. Since the ratios are equal, they form a proportion.

b. $\frac{6}{8}, \frac{24}{28}$

$\frac{6}{8} \stackrel{?}{=} \frac{24}{28}$	Write the equation.
$6(28) \stackrel{?}{=} 8(24)$	Find the cross products.
$168 \neq 192$	Simplify.

The cross products are not equal, so $\frac{6}{8} \neq \frac{24}{28}$. The ratios do not form a proportion.

In the proportion $\frac{0.4}{0.8} = \frac{0.7}{1.4}$ above, 0.4 and 1.4 are called the **extremes**, and 0.8 and 0.7 are called the **means**.

Key Concept — Means-Extremes Property of Proportion

- **Words** In a proportion, the product of the extremes is equal to the product of the means.
- **Symbols** If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.
- **Examples** Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

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SOLVE PROPORTIONS You can write proportions that involve a variable. To solve the proportion, use cross products and the techniques used to solve other equations.

Example 3 Solve a Proportion

Solve the proportion $\frac{n}{15} = \frac{24}{16}$.

$\frac{n}{15} = \frac{24}{16}$	Original equation
$16(n) = 15(24)$	Find the cross products.
$16n = 360$	Simplify.
$\frac{16n}{16} = \frac{360}{16}$	Divide each side by 16.
$n = 22.5$	Simplify.

The ratio of two measurements having different units of measure is called a **rate**. For example, a price of $1.99 per dozen eggs, a speed of 55 miles per hour, and a salary of $30,000 per year are all rates. Proportions are often used to solve problem involving rates.

Example 4 Use Rates

BICYCLING Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

Explore Let m represent the number of miles Trent can ride in 6 hours.

Plan Write a proportion for the problem.

$$\begin{matrix}\text{miles} \rightarrow \\ \text{hours} \rightarrow\end{matrix} \frac{30}{4} = \frac{m}{6} \begin{matrix}\leftarrow \text{miles} \\ \leftarrow \text{hours}\end{matrix}$$

Solve

$\frac{30}{4} = \frac{m}{6}$	Original proportion
$30(6) = 4(m)$	Find the cross products.
$180 = 4m$	Simplify.
$\frac{180}{4} = \frac{4m}{4}$	Divide each side by 4.
$45 = m$	Simplify.

Examine If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride 6×7.5 or 45 miles. The answer is correct.

Since the rates are equal, they form a proportion. So, Trent can ride 45 miles in 6 hours.

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Crater Lake is a volcanic crater in Oregon that was formed by an explosion 42 times the blast of Mount St. Helens.

Source: travel.excite.com

Example 5 Use a Scale Drawing

CRATER LAKE The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about $1\frac{3}{4}$ inches. What is the distance between these two places?

Let d represent the actual distance.

$$\begin{matrix}\text{scale} \rightarrow \\ \text{actual} \rightarrow\end{matrix} \frac{2}{9} = \frac{1\frac{3}{4}}{d} \begin{matrix}\leftarrow \text{scale} \\ \leftarrow \text{actual}\end{matrix}$$

$2(d) = 9\left(1\frac{3}{4}\right)$	Find the cross products.
$2d = \frac{63}{4}$	Simplify.
$2d \div 2 = \frac{63}{4} \div 2$	Divide each side by 2.
$d = \frac{63}{8}$ or $7\frac{7}{8}$	Simplify.

The actual distance is about $7\frac{7}{8}$ miles.

Check for Understanding

Concept Check

1. **OPEN ENDED** Find an example of ratios used in advertisements.
2. **Explain** the difference between a ratio and a proportion.
3. **Describe** how to solve a proportion if one of the ratios contains a variable.

Guided Practice

Use cross products to determine whether each pair of ratios form a proportion. Write *yes* or *no*.

4. $\frac{4}{11}, \frac{12}{33}$
5. $\frac{16}{17}, \frac{8}{9}$
6. $\frac{2.1}{3.5}, \frac{0.5}{0.7}$

Solve each proportion. If necessary, round to the nearest hundredth.

7. $\frac{3}{4} = \frac{6}{x}$
8. $\frac{a}{45} = \frac{5}{15}$
9. $\frac{0.6}{1.1} = \frac{n}{8.47}$

Application

10. **TRAVEL** The Lehmans' minivan requires 5 gallons of gasoline to travel 120 miles. How much gasoline will they need for a 350-mile trip?

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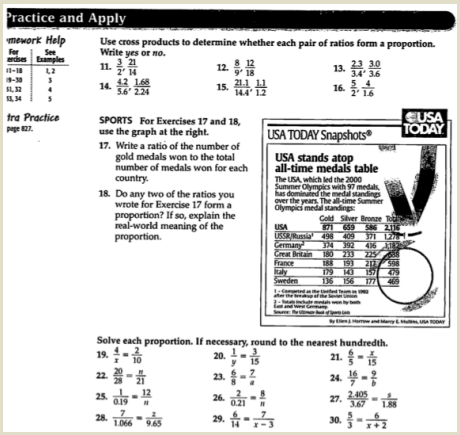

Practice and Apply

Use cross products to determine whether each pair of ratios forms a proportion. Write *yes* or *no*.

11. $\frac{3}{2} \stackrel{?}{=} \frac{21}{14}$
12. $\frac{8}{9} \stackrel{?}{=} \frac{12}{18}$
13. $\frac{2.3}{3.4} \stackrel{?}{=} \frac{3.0}{3.6}$
14. $\frac{4.2}{5.6} \stackrel{?}{=} \frac{1.68}{2.24}$
15. $\frac{21.1}{14.4} \stackrel{?}{=} \frac{1.1}{1.2}$
16. $\frac{5}{2} \stackrel{?}{=} \frac{4}{1.6}$

SPORTS For Exercises 17 and 18, use the graph at the right.

17. Write a ratio of the number of gold medals won to the total number of medals won for each country.
18. Do any two of the ratios you wrote for Exercise 17 form a proportion? If so, explain the real-world meaning of the proportion.

USA TODAY Snapshots®

USA stands atop all-time medals table

The USA, which led the 2000 Summer Olympics with 97 medals, has dominated the medal standings over the years. The all-time Summer Olympics medal standings:

	Gold	Silver	Bronze	Total
USA	871	659	586	2,116
USSR/Russia[1]	498	409	371	1,278
Germany[2]	374	392	416	1,182
Great Britain	180	233	225	638
France	188	193	212	598
Italy	179	143	157	479
Sweden	136	156	177	469

Solve each proportion. If necessary, round to the nearest hundredth.

19. $\frac{4}{x} = \frac{2}{10}$
20. $\frac{1}{y} = \frac{3}{15}$
21. $\frac{6}{5} = \frac{x}{15}$
22. $\frac{20}{28} = \frac{n}{21}$
23. $\frac{6}{8} = \frac{7}{a}$
24. $\frac{16}{7} = \frac{9}{b}$
25. $\frac{1}{0.19} = \frac{12}{n}$
26. $\frac{2}{0.21} = \frac{8}{n}$
27. $\frac{2.405}{3.67} = \frac{s}{1.88}$
28. $\frac{7}{1.066} = \frac{z}{9.65}$
29. $\frac{6}{14} = \frac{7}{x-3}$
30. $\frac{5}{3} = \frac{6}{x+2}$

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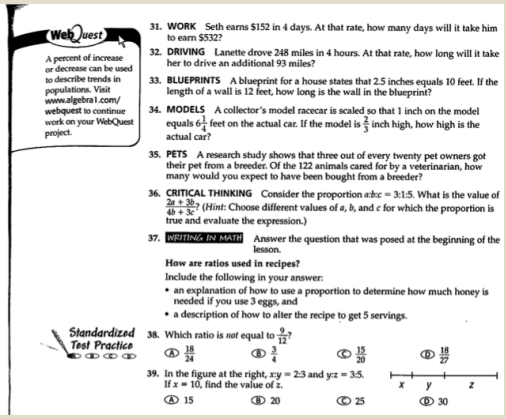

WebQuest

A percent of increase or decrease can be used to describe trends in populations. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

31. **WORK** Seth earns $152 in 4 days. At that rate, how many days will it take him to earn $532?
32. **DRIVING** Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?
33. **BLUEPRINTS** A blueprint for a house states that 2.5 inches equals 10 feet. If the length of a wall is 12 feet, how long is the wall in the blueprint?
34. **MODELS** A collector's model racecar is scaled so that 1 inch on the model equals $6\frac{1}{2}$ feet on the actual car. If the model is $\frac{2}{3}$ inch high, how high is the actual car?
35. **PETS** A research study shows that three out of every twenty pet owners got their pet from a breeder. Of the 122 animals cared for by a veterinarian, how many would you expect to have been bought from a breeder?
36. **CRITICAL THINKING** Consider the proportion $a:b:c = 3:1:5$. What is the value of $\frac{2a+3b}{4b+3c}$? (*Hint*: Choose different values of a, b, and c for which the proportion is true and evaluate the expression.)
37. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are ratios used in recipes?

Include the following in your answer:

- an explanation of how to use a proportion to determine how much honey is needed if you use 3 eggs, and
- a description of how to alter the recipe to get 5 servings.

Standardized Test Practice

38. Which ratio is *not* equal to $\frac{9}{12}$?
Ⓐ $\frac{18}{24}$ Ⓑ $\frac{3}{4}$ Ⓒ $\frac{15}{20}$ Ⓓ $\frac{18}{27}$

39. In the figure at the right, $x:y = 2:3$ and $y:z = 3:5$. If $x = 10$, find the value of z.
Ⓐ 15 Ⓑ 20 Ⓒ 25 Ⓓ 30

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Benefits

- Variety
- Reaching different types of learners
- Deeper student understanding

Deeper teacher understanding!

15

Benefits

- Prioritizing topics
- Moving topics within a course
- Moving topics between courses
- Eliminating topics?

16

How?

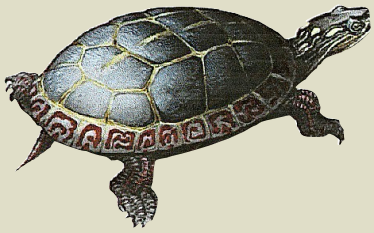
- Teacher collaboration is the engine
- Set priorities! / Be realistic
- Create a teacher culture of constant evaluation
- Schedule some summer work
- Go back and forth: pedagogy and curriculum

17

A lot of work!

18

Patience



Slow is fast,
and fast is slow!

19

Collaboration

Ideally, collaboration
with colleagues at school,
but why not with others?

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www.EscapeTheTextbook.org
www.edWeb.net/escape

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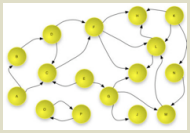
Online network



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In-person meetings
(in the Bay Area)

◊ Do math together

◊ Discuss pedagogy

◊ Share activities

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Conference

◊ Do math together

Paul Zeitz, University of San Francisco
author of *The Art and Craft of Problem Solving*
"Games, investigation, and problem solving"

24

Conference

◊ Discuss pedagogy

Jo Boaler, Stanford University
author of *What's Math Got To Do With It?*
"The Many Colors of Algebra – Engaging Disaffected Students Through Collaboration and Agency."

25

Conference

◊ Share activities

- Middle School
- Algebra 1
- Geometry
- Algebra 2 / Precalc
- Precalc / Calculus
- Statistics

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Conference

Saturday, February 12

8:45 am to 3:00 pm

at the Urban School of San Francisco

1563 Page St, SF, 94117

$25

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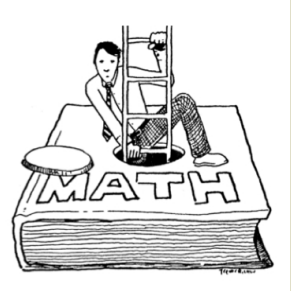

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Conference info and registration:

www.EscapeTheTextbook.org

Online network:

www.edWeb.net/escape