

# Representing Functions

## Teacher's Notes

**Core Sequence:** 1-23

**Suitable for homework:** 5-23

**Useful for Assessment:** 3, 5, 9-10, 15, 18-19, 22-23

### What this Lesson is About:

- Review of linear functions
- Review of linear function foci in function diagrams
- Review of simultaneous equations
- Families of linear functions

We used function diagrams to introduce functions earlier on, and we use them again, this time to review basic ideas about linear functions, and to point out an interesting duality between points and lines. Some students will appreciate the deeper, more theoretical ideas in the lesson, but everyone will benefit from the opportunity to review basic ideas in a fairly easy lesson.

### Points and Lines

#### Focus on Simultaneous Equations

We first encountered the focus in **Focus on Function Diagrams**. As it turns out, there is a one-to-one correspondence between foci and linear functions with  $m \neq 1$ . Each function has a unique focus, and each focus corresponds to a unique function.

For #1, students should have no trouble finding the slope and intercept from the graph, but for #4 they may not think of looking for the  $m$  and  $b$  parameters directly in the diagram. After they have found the equations, it would be interesting to compare strategies.

The most effective way is to find the in-out line through the focus and the point 0 on the  $x$  number line. That will give the  $y$ -intercept  $(0, -2)$ , so  $b = -2$ . Then one can look for the magnification  $m$  by looking for one other in-out line, preferably one with integral coordinates, and finding the ratio between the change in  $y$  values and the change in  $x$  values. Though other choices are possible, the in-out line  $(-3, -3)$  is convenient. It allows us to see that when  $x$  changes by 3,  $y$  changes by 1. Therefore the magnification must be  $1/3$ . (However, you may hold off on presenting this approach until after having done the next section, where the function diagram representation of  $m$  and  $b$  is reviewed.)

#6-10 review familiar material in a new setting.

#3, #5, #9 and the closing paragraphs of both sections address the point-line duality that is inherent to these representations. The idea of point-line duality is not important for students to master at this level, but those who appreciate this aesthetically should be encouraged to stick with mathematics. Duality is fundamental to projective geometry, a branch of mathematics they will have a chance to study in college.

## Families of Functions

You may use the idea in #26 as a motivation in the real world for the concept of families of functions. However we concentrate on  $m$  and  $b$  families, in order to review once again the significance of those parameters.

To speed up the making of the function diagrams in this section, remind the students that all they need is to find the focus, which can be done by sketching two in-out lines. (A third one can be used as a check.)

#18 reviews the previous section, since all systems where both equations are of the form  $y = mx - 2$  will have  $(0, -2)$  for a solution.

When discussing  $m$  families, you may have a class discussion of how the position of the focus is related to the value of  $m$ , and of how  $m$  is the magnification from the  $x$  to the  $y$  number line. (See **Focus on Function Diagrams.**)

Note that #24 adds an important footnote by reminding students that some linear functions do not have a focus, because their in-out lines are parallel. Alternately, for those functions, we think of the focus being a point *at infinity*, which makes it possible to preserve the duality. (All the points at infinity make up the line at infinity.)

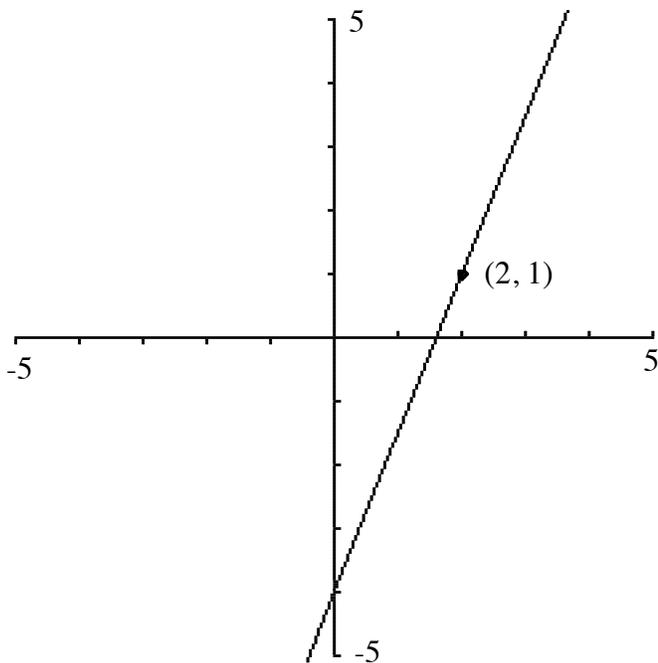
## Representing Functions

**You will need:** graph paper, function diagram paper

### Points and Lines

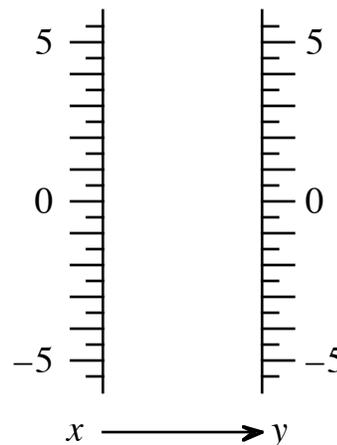
As you know, an  $(x, y)$  pair is represented as a point on a Cartesian graph, and as an in-out line on a function diagram. In this section, we will review how an equation of the form  $y = mx + b$  is represented in these formats.

1. For the function represented by this Cartesian graph:
  - a. write the equation
  - b. draw a function diagram



2. Extend the in-out lines in the function diagram you made in the previous problem. They should meet in one point, called the focus.
3. € What is the minimum number of lines one needs to draw to find the focus? Explain.

Actually, a function of the form  $y = mx + b$  can be represented by just the focus, as you will see in the next problem.



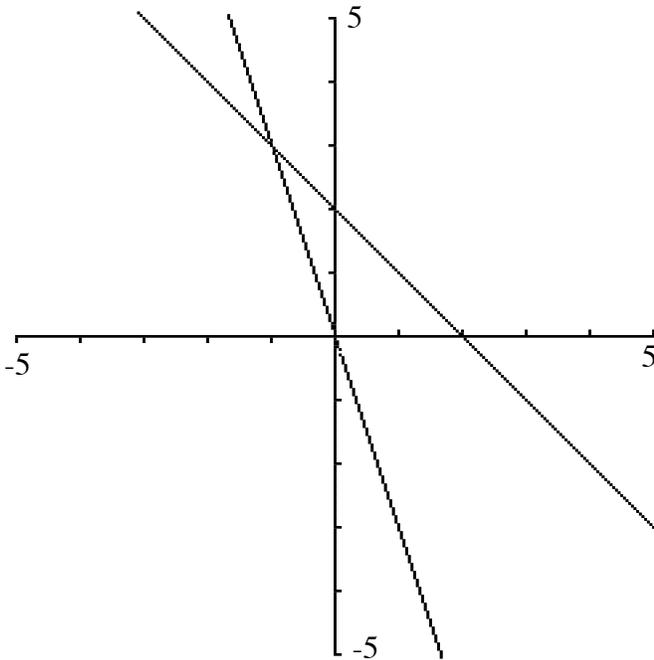
4. The figure shows the focus of a certain function of the form  $y = mx + b$ .
  - a. Place a ruler on the focus, and find three in-out lines. Do not draw the lines, but keep a record of the  $(x, y)$  pairs.
  - b. Find the equation.
5. € If you were to make a Cartesian graph of this function, what is the minimum number of points you would need to plot? Explain.

The table below shows how points and lines appear in the two representations. Notice how points and lines are switched when going from one representation to the other.

Object	Representation	
	on Cartesian graph	on function diagram
$(x,y)$ pair	one point	one line (in-out)
linear equation	one line	one point (the focus)

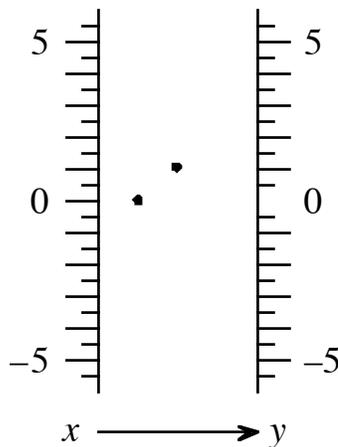
### Focus on Simultaneous Equations

The graph shows  $y = -3x$  and  $y = -x + 2$ .



6. What point do the two lines have in common?

The function diagram shows the foci of  $y = -3x$  and  $y = -x + 2$ . (Foci is the plural of focus.)



7. Check that the foci are placed correctly:
  - a. Place a ruler on the focus and find three in-out lines for each function. Do not draw the lines, but keep a record of the  $(x, y)$  pairs.
  - b. Check that the  $(x, y)$  pairs you found satisfy the equations.
8. If one were to draw an in-out line containing both foci, what  $(x, y)$  pair would it represent?
9. € How is the solution of a system of linear equations represented on:
  - a. a Cartesian graph?
  - b. a function diagram?
10. € Explain how to solve this system by using a function diagram. (Hint: First find each focus, then find the solutions.)

$$\begin{cases} y = .5x + 2 \\ y = 2x - 1 \end{cases}$$

Because two lines meet in a point, the solution to a system of simultaneous equations is represented on a Cartesian graph by a point. Because two points determine a line, the solution to a system of linear equations is represented on a function diagram by a line (an in-out line).

## Families of Functions

**Definition:** A *family* of functions is a group of functions that share a certain attribute.

11. All functions with equations of the form  $y = 5x + b$  belong to the  $m = 5$  family.
- Sketch the graphs of two members of the family.
  - What do all graphs for this family have in common?
12. All functions with equations of the form  $y = mx + 7$  belong to the  $b = 7$  family.
- Sketch the graphs of two members of the family.
  - What do all graphs for this family have in common?

All functions in the same  $b$ -family have foci that lie on the same in-out line.

13. These four functions are in the same  $b$ -family. For each one, draw in-out lines to find the focus, and mark it with a colored pen or pencil. Do all four on the same diagram.

a. $y = 0.5x - 2$	b. $y = 2x - 2$
c. $y = -2x - 2$	d. $y = -0.5x - 2$

14. What is the family name for the functions in #13?
15. € Why do all the foci of the functions in #13 lie on the same in-out line. Which in-out line is it? Explain.
16. The foci for all functions in the family  $b = -3$  also lie on one in-out line. Which line? Explain how you know.

Many  $m$ -families also have foci that lie on the same in-out line in a function diagram.

17. a. On a function diagram, find and mark the focus for  $y = -2x + 3$ .
- b. On the same function diagram, find and mark the focus for  $y = -2x + 1$ .
- c. Find and mark the focus for several other graphs of the form  $y = -2x + k$ .
18. € What is the family name for all the functions in the previous problem? Explain why the foci are all on the same line. Describe the line.
19. € If two functions both have a focus on the same vertical line, what would their Cartesian graphs have in common?
20. \* What is the family name for all functions with focus half-way between the two number lines?
21. There is one  $m$  family for which the function diagrams have no focus, because the in-out lines do not meet. Which  $m$  family is this?
22. **Summary.** On a function diagram, what is true of the foci of all linear functions in the same
- $m$  family?
  - $b$  family?