Connecting the Dots

Geoboard Area

Henri Picciotto
henri@MathEducationPage.org
www.MathEducationPage.org

Lessons from

Geometry Labs

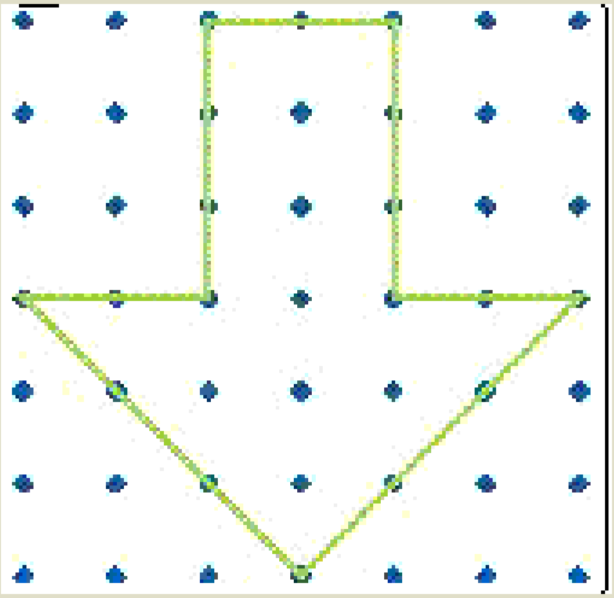
Henri Picciotto

(free download at www.MathEducationPage.org)

as well as...

interesting related unsolved problems

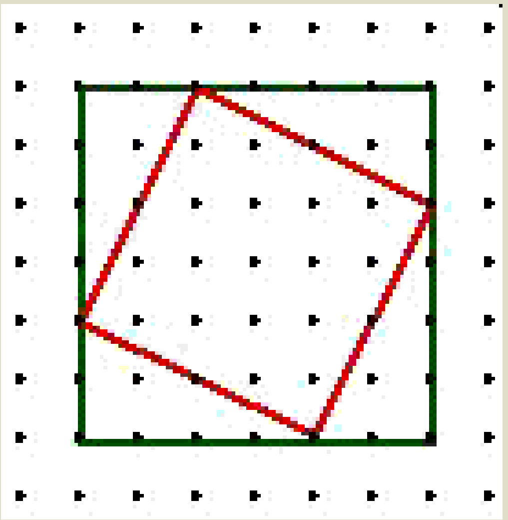
Working on a lattice



Find geoboard shapes with area 15

Find as many geoboard squares of different sizes (and their areas) as you can.

Hint: there are more than 10



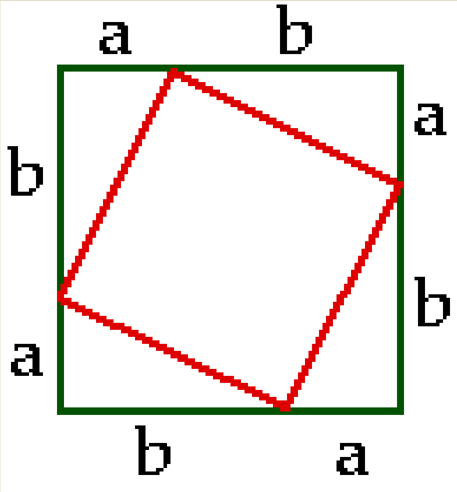
What is the area of the red square?

outer square: 6^2

each triangle: $\frac{2 \cdot 4}{2} = 4$

inner square: $36 - 4 \cdot 4 = 20$

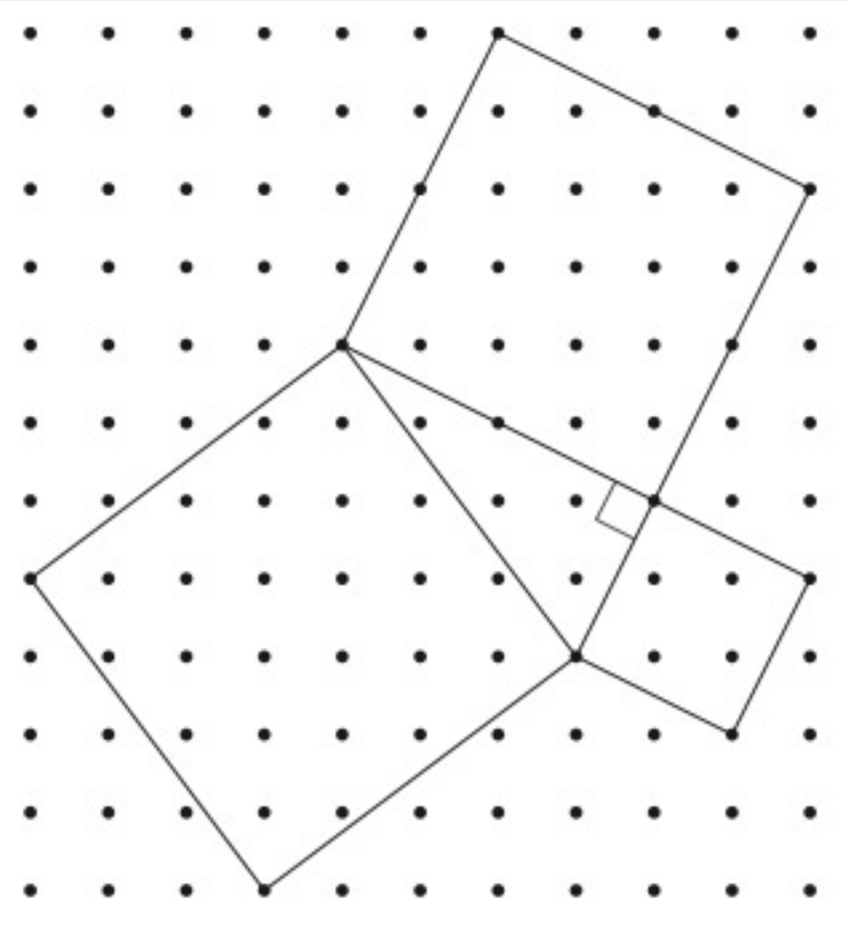
Generalize



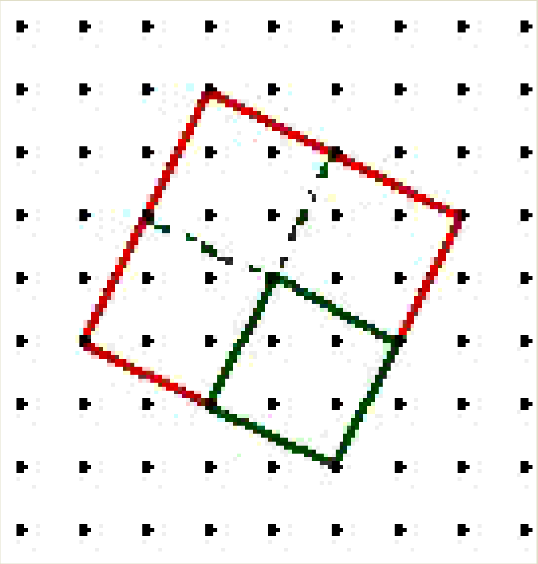
outer square: $(a + b)^2$

each triangle: $\frac{a \cdot b}{2}$

inner square: $(a + b)^2 - 2ab = a^2 + b^2$



Area of squares		
Small	Medium	Large



Area of the red square: 20

Area of the green square: 5

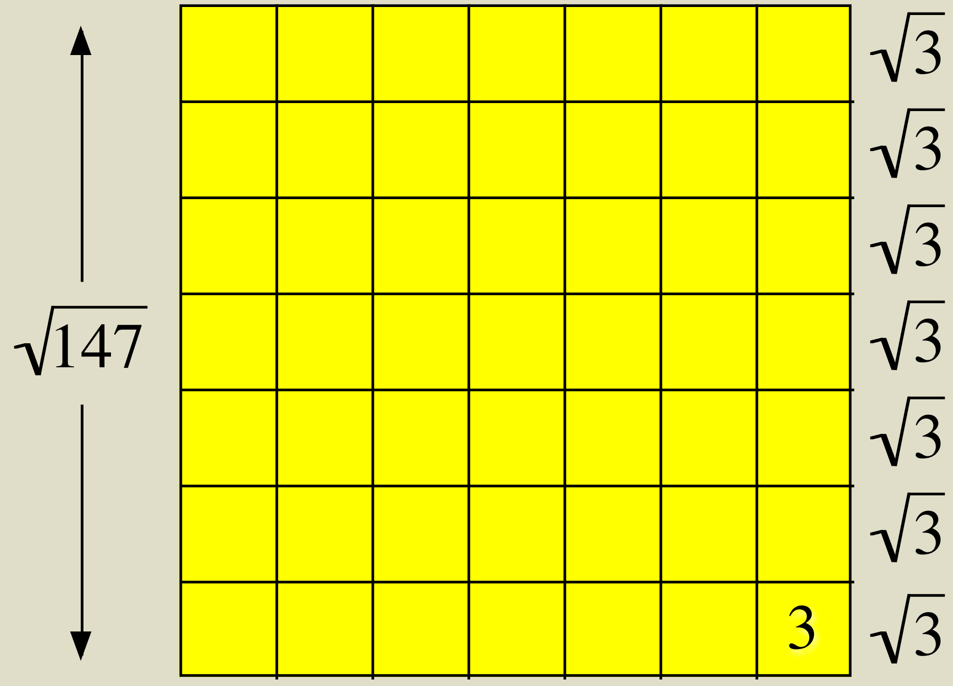
Side of the red square: $\sqrt{20}$

Side of the green square: $\sqrt{5}$

Conclusion: $\sqrt{20} = 2\sqrt{5}$!

Simplify: $\sqrt{147}$

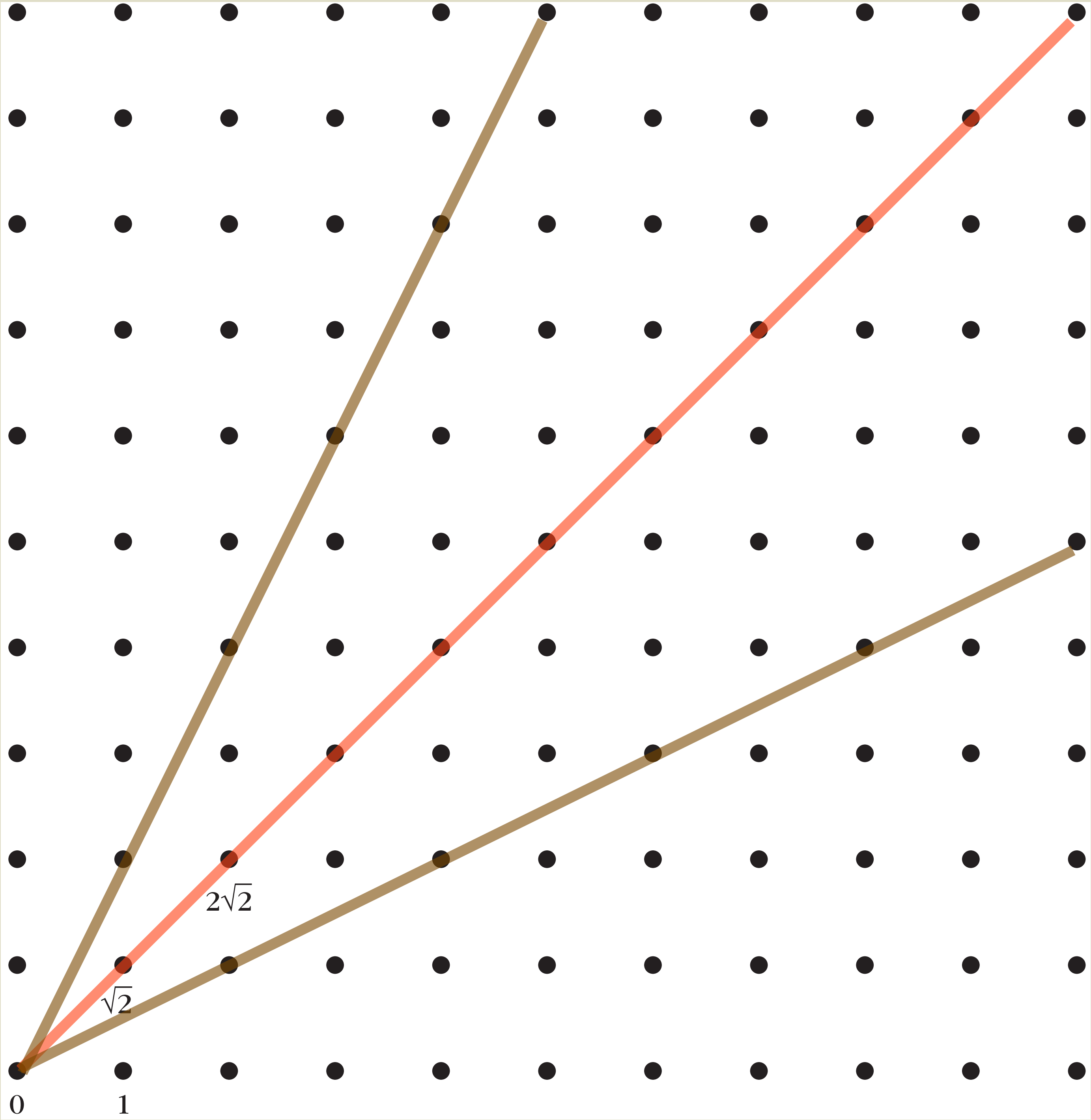
Area: 147



$$\sqrt{147} = \sqrt{49 \cdot 3} = 7\sqrt{3}$$

Distance to the origin

1. What is the distance from each geoboard peg to the origin? Write your answers in simple radical form on the figure below.



Discussion

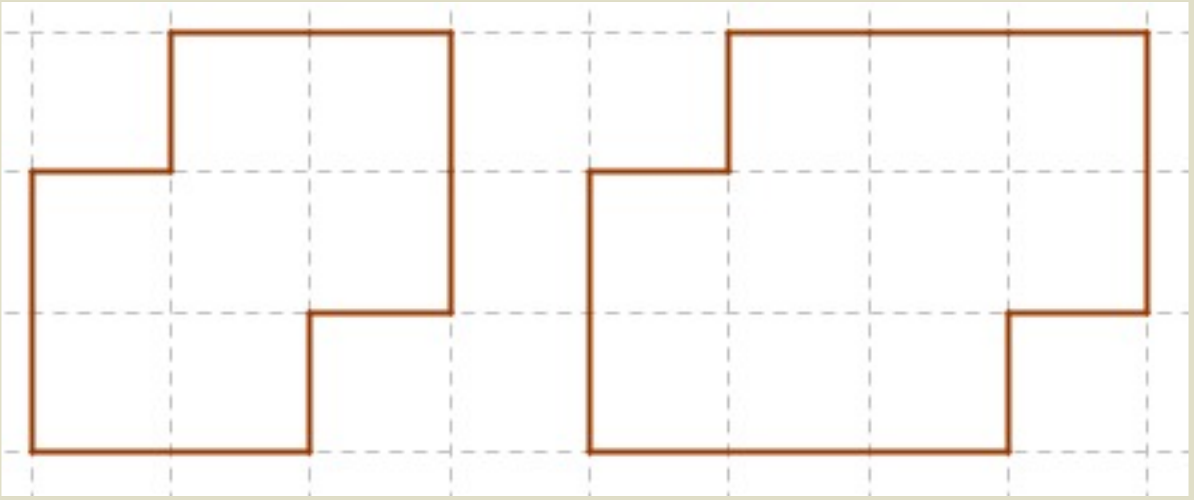
A. Discuss any patterns you notice in the distances. Use color to highlight them on the figure. In particular, refer to the following features.

a. Symmetry

b. Slope

Puzzle

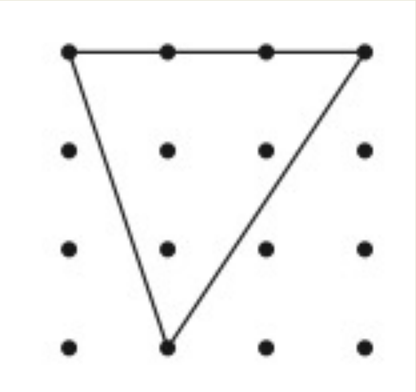
Draw a polygon *following grid paper lines*.
No crossings, no holes.

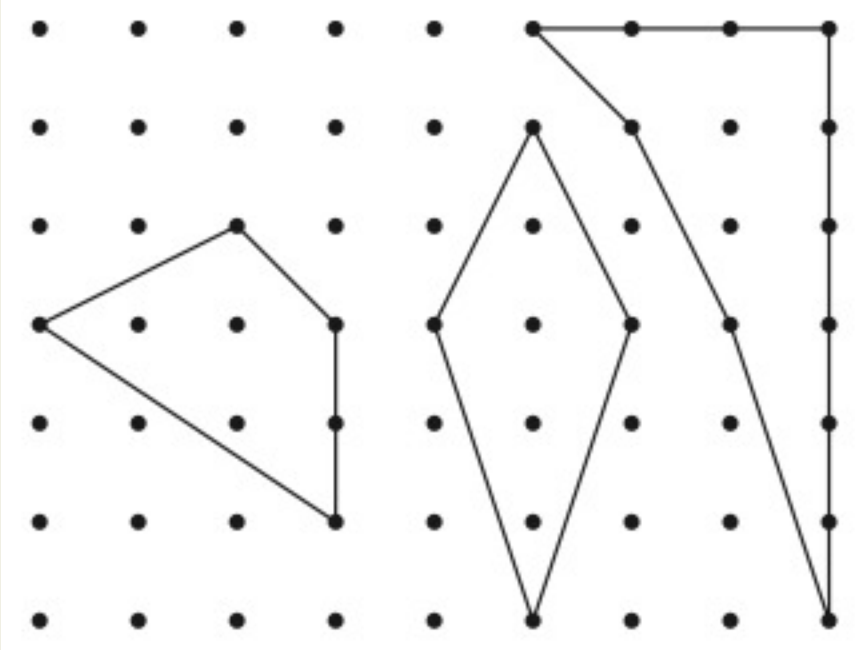


Now inscribe a square in it, with all its vertices
at lattice points on the perimeter of the polyomino.

Conjecture: it is impossible to draw a polyomino
that does ***not*** have such a square inscribed in it.

Pick's Formula

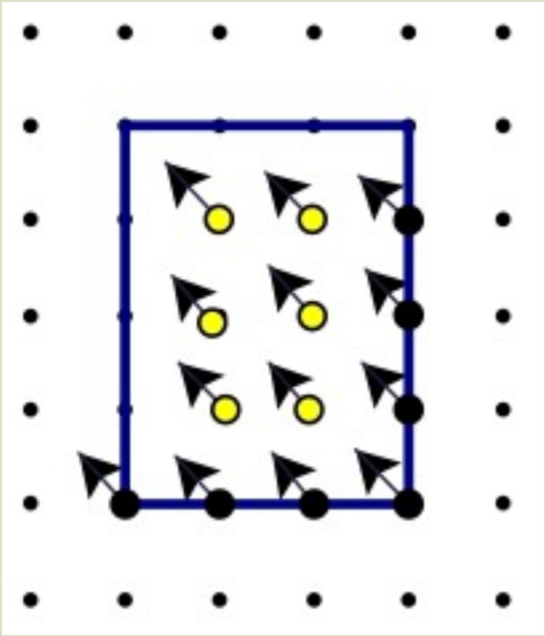


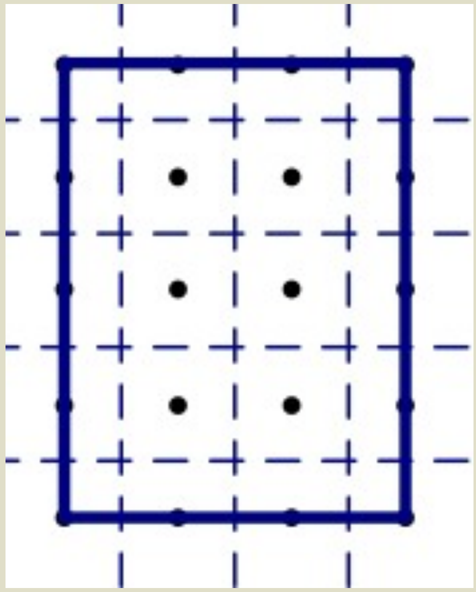


Inside dots	Bound. dots	Area

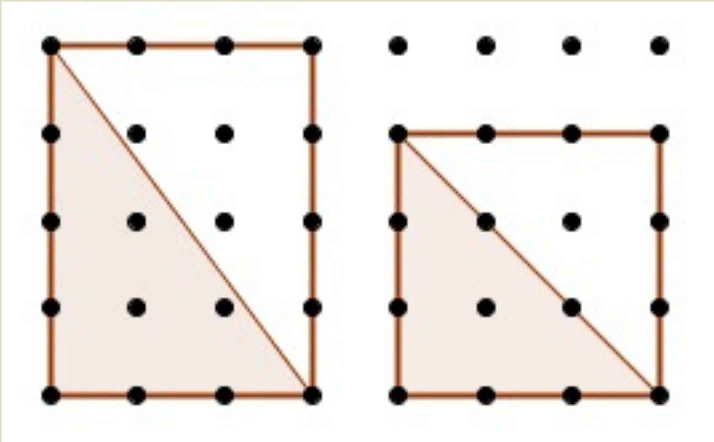
Proving Pick

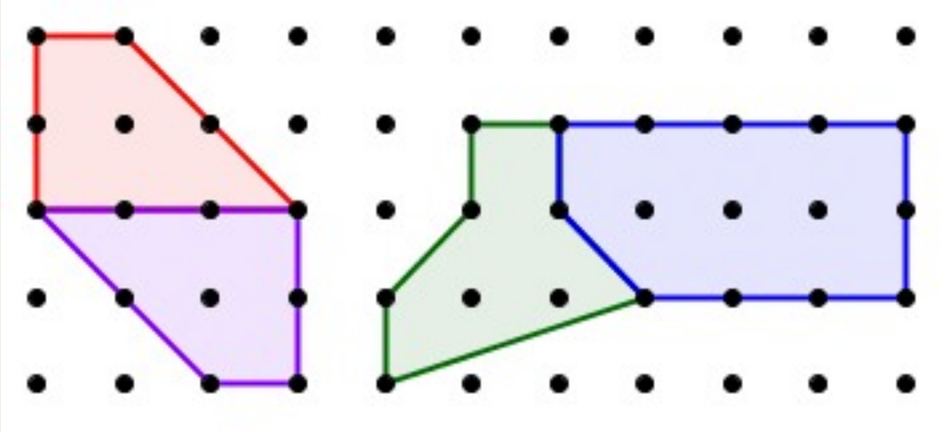
$$A = i + \frac{b}{2} - 1$$

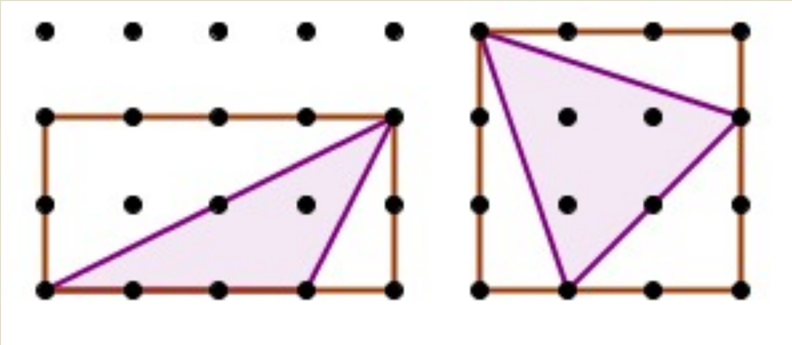




The rest of the proof (outline)



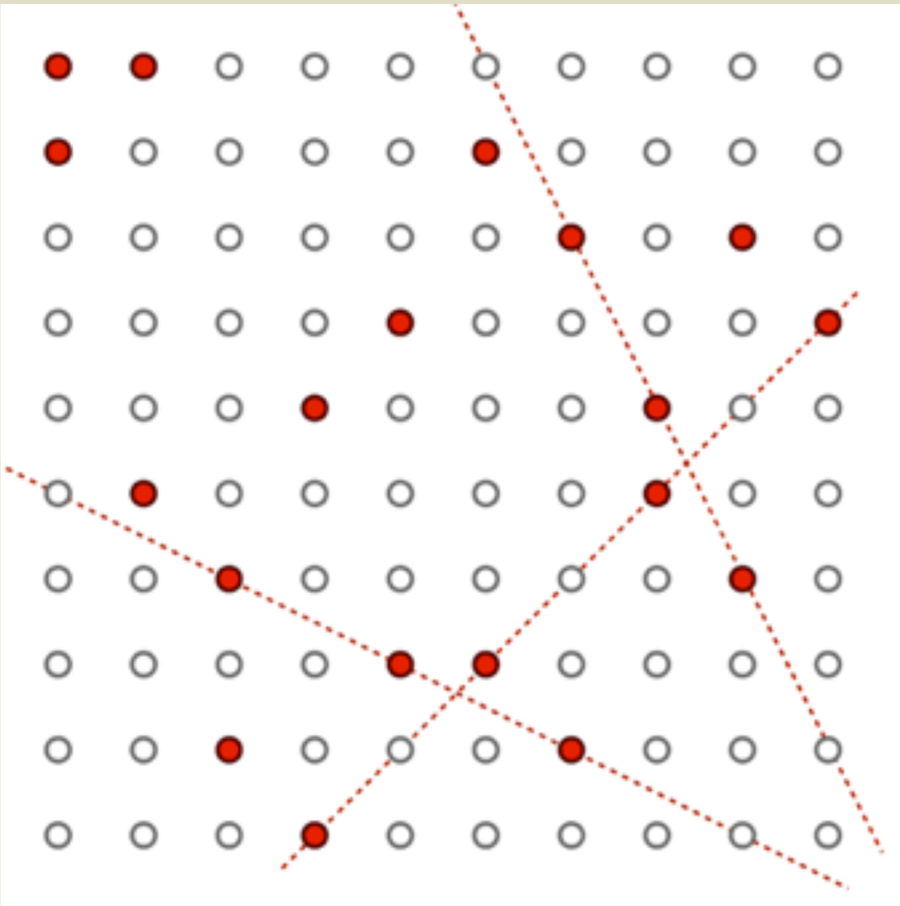


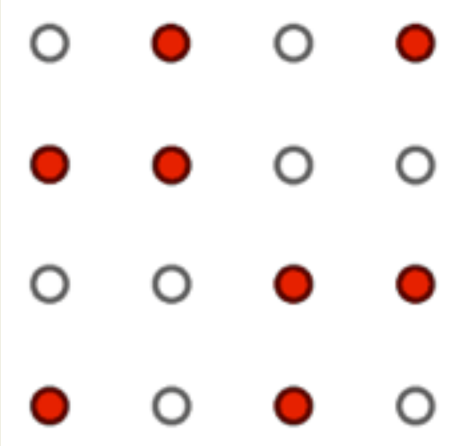


www.MathEducationPage.org/geometry-labs/pick/

No Three in a Row

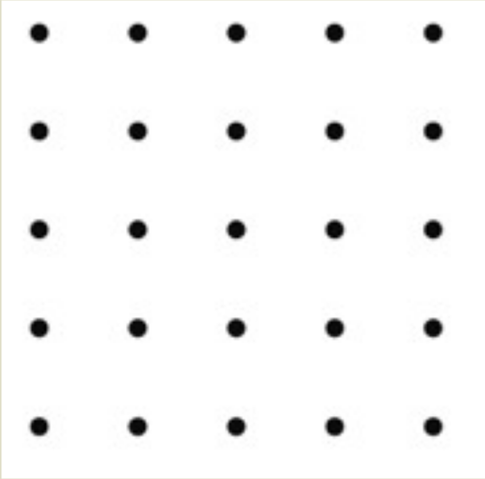
Consider an n by n lattice.
Is it always possible to choose 2n points in it
so that no three points are in a line?



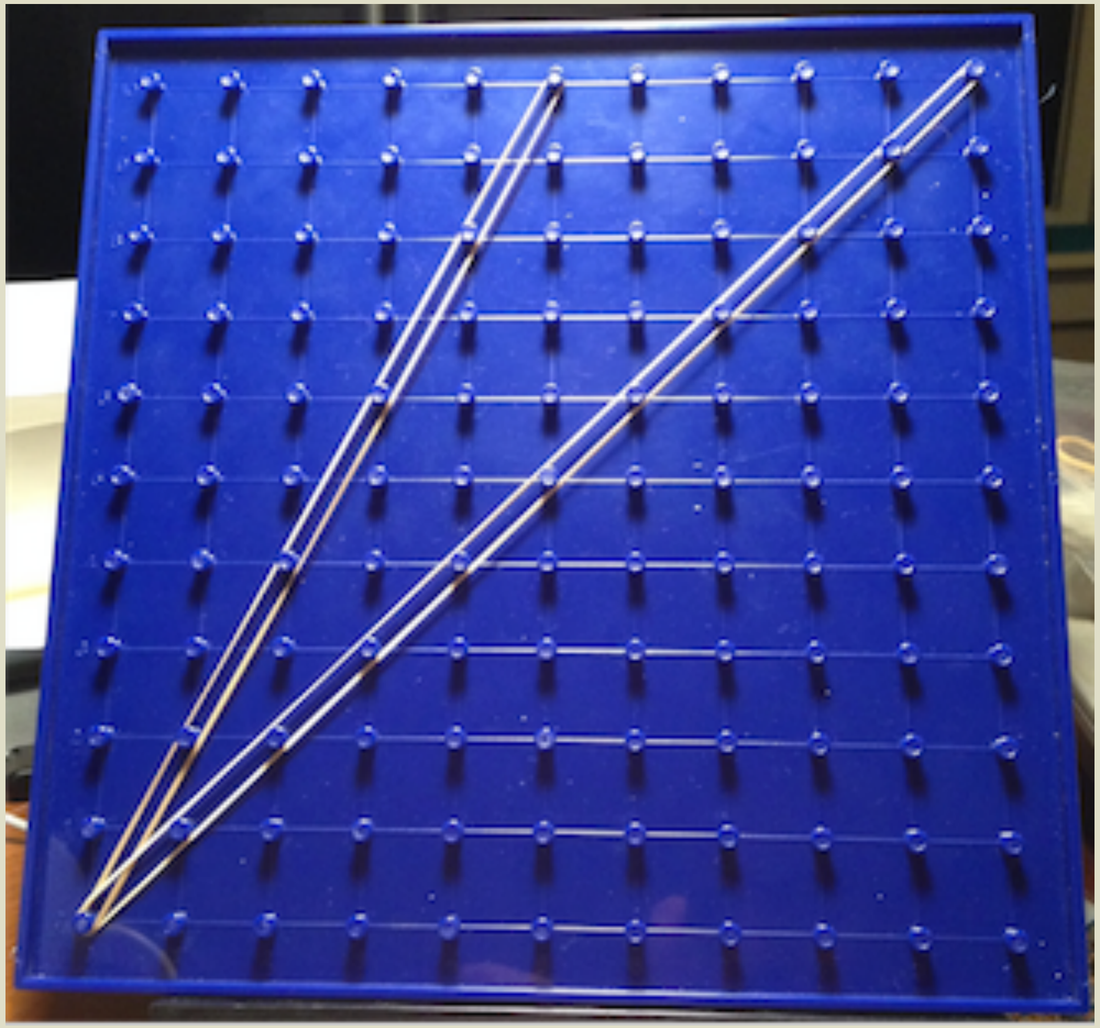


Challenge

10 points on a 5 by 5 lattice,
no three in a row

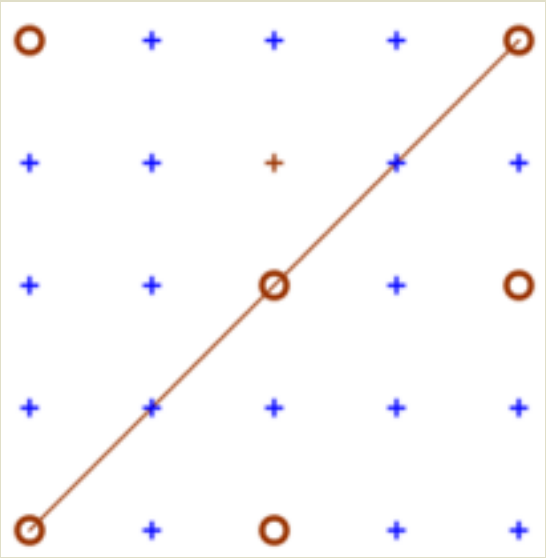


Geoboard Slope



Heilbronn's Triangle

Place six points in a 5 by 5 array so that the triangle of *least* area has as *great* an area as possible



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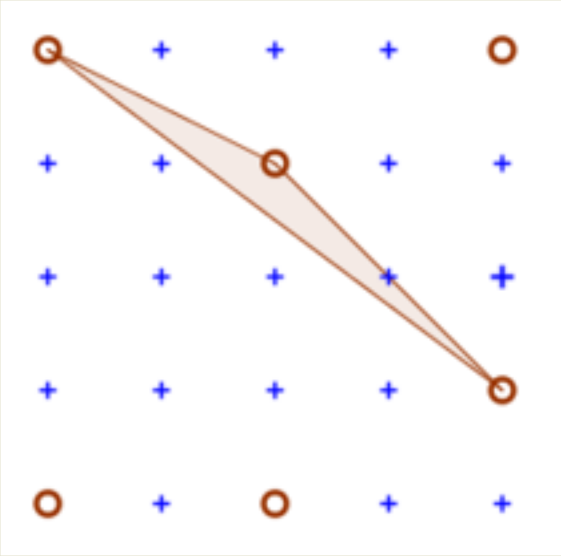

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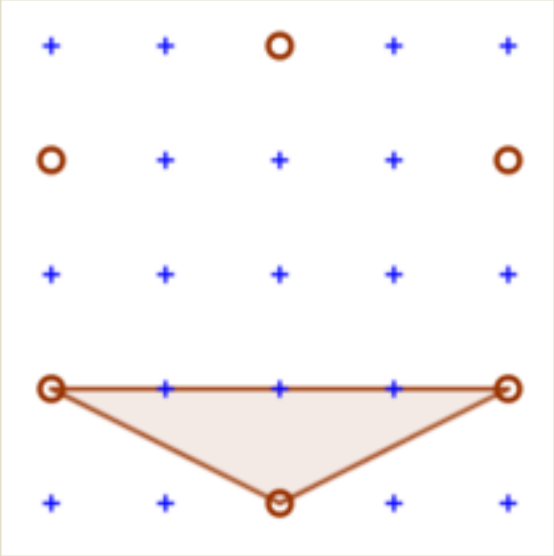
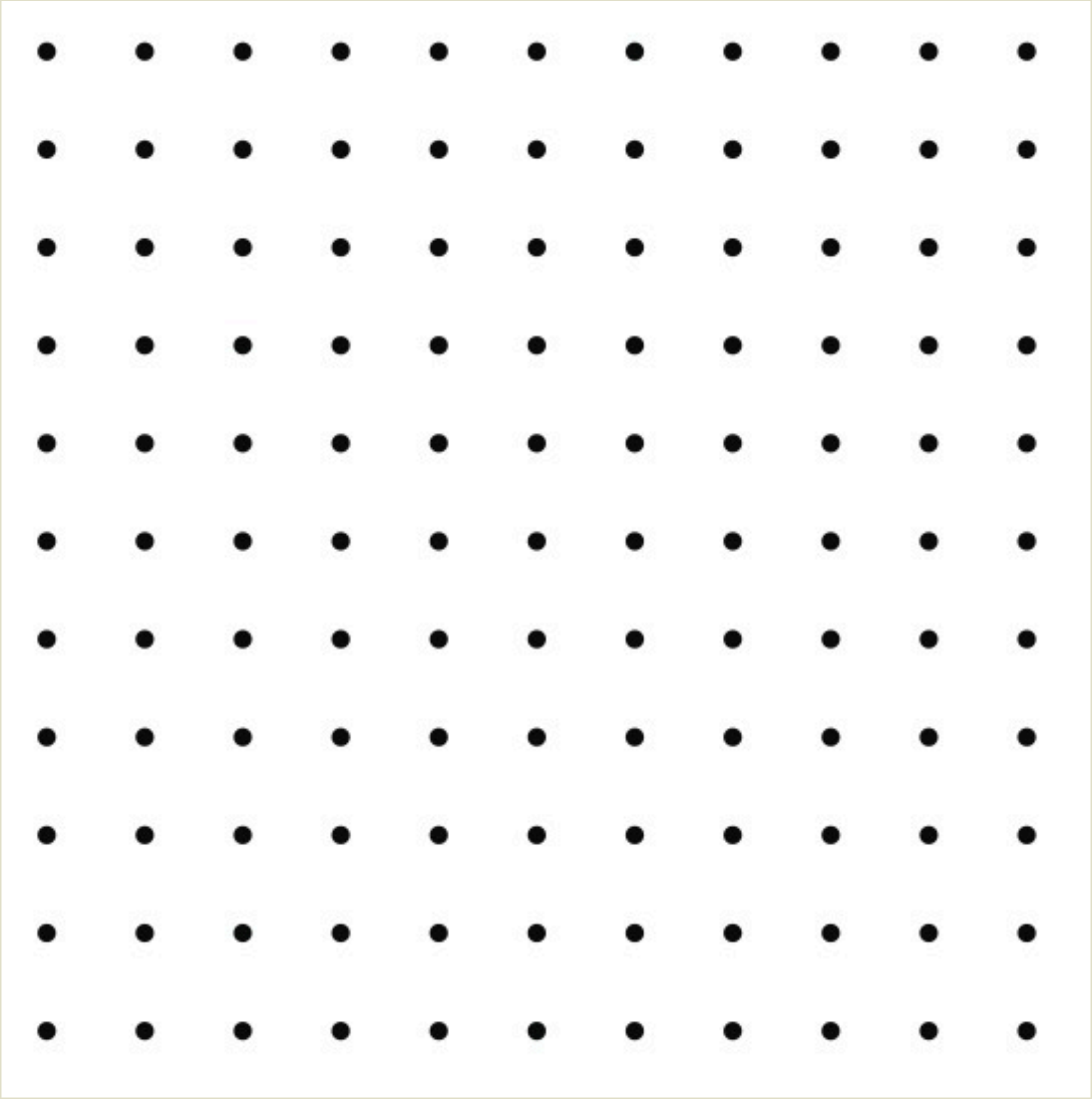


2

Challenge:
n points in an 11 by 11 array



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