Geoboard Math

The geoboard provides a nicely constrained environment for mathematical exploration, and lends itself to many activities where students (and teachers!) can discover or apply important ideas in the K-12 curriculum.

11 × 11 Geoboard

Geoboard Area

Ask your students to use rubber bands to make geoboard shapes that have a given area. Encourage them to try to find figures other than the most obvious ones by suggesting they find shapes different from those of their neighbors. Hold up the most interesting solutions for the whole class to see.

Discuss how to find the areas of rectangles and right triangles that have horizontal and vertical sides. Once students understand how to find those areas, they can find the area of any geoboard shape by adding and/or subtracting the areas of rectangles and right triangles.

Geoboard Squares

Tell your students that there are 33 different-sized squares on the geoboard.Ask them to find them and to find the area of each one. (Note that 23 of the squares are "tilted.") It is not actually necessary to find all the squares: the point of the lesson, aside from the fact that it develops students' visual sense, is that finding the area of the squares is a rehearsal of an accessible proof of the Pythagorean theorem.

Pick's Formula

Have your students find the formula that relates the number of pegs inside a geoboard figure, the number of pegs on its perimeter (boundary), and the area of the figure.

Teacher Challenges

1. Find all the **isosceles geoboard triangles** whose base is not parallel to or at a 45° angle from the edge of the board.

2. Find all **triangles with area 15**, such that no side is parallel to the edge of the board. (They don't need to be isosceles.)

3. For a given set of lattice points, any three determine a triangle (possibly a "flat" triangle with area 0.) Among those triangles, call a triangle with least area a *Heilbronn triangle*. Choose k lattice points to **maximize the area of the Heilbronn triangle**. For example, for $k = 6$, it is possible to arrange six dots and get a Heilbronn triangle with area > 1.
   Find the optimal arrangement for k = 4, 5, 6, and 7 on an 11 by 11 lattice.

CircleTrig Geoboard

Angles and Triangles in a Circle

Use this lesson to review the triangle sum theorem, the isosceles triangle theorem, and the exterior angle theorem.You can also use it as preparation for introducing the inscribed angle theorem and the special case of angles inscribed in a half-circle (Thales's theorem).

Ask students to make triangles on the geoboard with *one vertex at the circle center and the other two on the circle*. For each triangle, ask them to find the measurements of all the angles. They should look for each of the following eight types of triangles: equilateral, right isosceles, acute scalene, half-equilateral (aka 30-60-90), acute isosceles, obtuse isosceles, right scalene, obtuse scalene.

Repeat the activity, but this time ask the students to make *inscribed triangles, with one side going through the center of the circle*. If they have trouble finding the angles, you can give this hint: "when working with circles, listen to the radii".

Finally, you can ask students to make *inscribed triangles such that the center of the circle is inside or outside the triangle.* Finding all the angles guides students through a rehearsal of the proof of the inscribed angle theorem.

Introduction to Trigonometry

The CircleTrig geoboard can be used to find the sine, cosine, and tangent of any angle accurate to two significant digits.You can also read the inverse functions on the board. In each case, you'll read the desired value by identifying the tick mark that lies between the two lengths of a stretched rubber band.

One way to reinforce students' understanding of the meaning of these ratios is to ask them to solve right-triangle problems, perhaps presented as word problems, without the help of calculators or tables. Using the CircleTrig board as a tool should help cement the geometric meaning of sine, cosine, and tangent.

A possible follow-up is to make a table of values for the ratios, perhaps at 15 degree intervals, and look for patterns, for example the relationship between different trigonometric ratios for complementary angles.

More About the Geoboard

All these activities are from *Geometry Labs,* a free download from my Web site (MathEducationPage.org). For a lot more information on geoboards, go to:
https://www.mathedpage.org/geoboard

-- Henri Picciotto