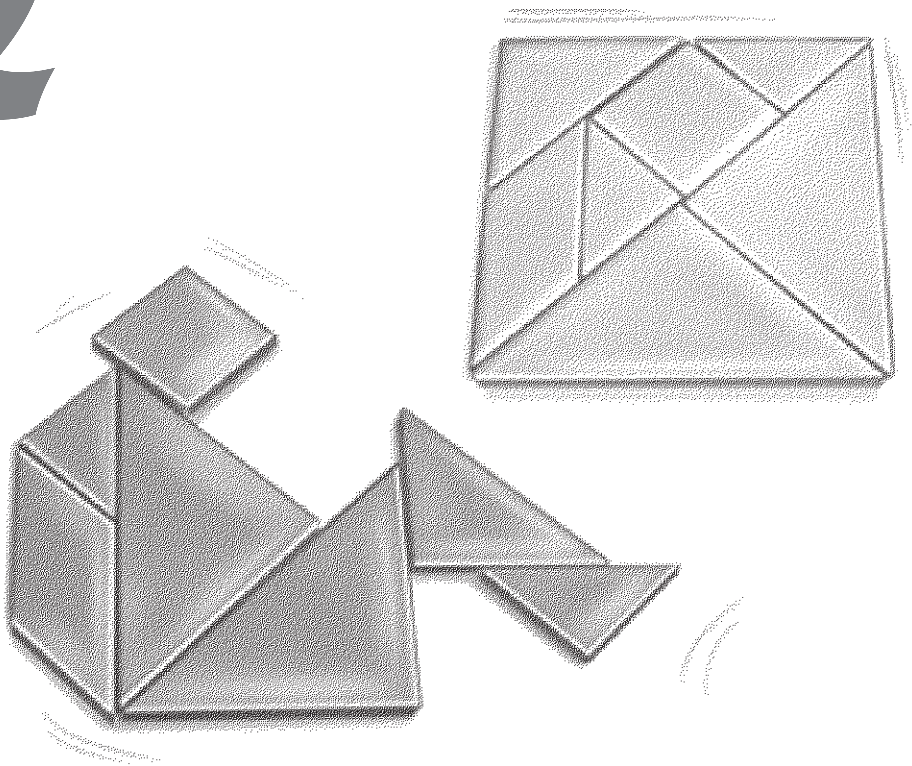
2 Tangrams



Tangram puzzles are quite accessible, and they help develop students' feel for the ubiquitous isosceles right triangle. This short section introduces some basic vocabulary and concepts about polygons in a context drawn from recreational mathematics. In addition, other concepts that will return in later sections make their first appearance here: square roots, similarity, symmetry, and convexity.

For more on tangrams, see:

Amusements in Mathematics, by Henry Dudeney (Dover Books)

Time Travel and Other Mathematical Bewilderments, by Martin Gardner (W. H. Freeman & Co.)

The latter book also features a bibliography about tangrams.

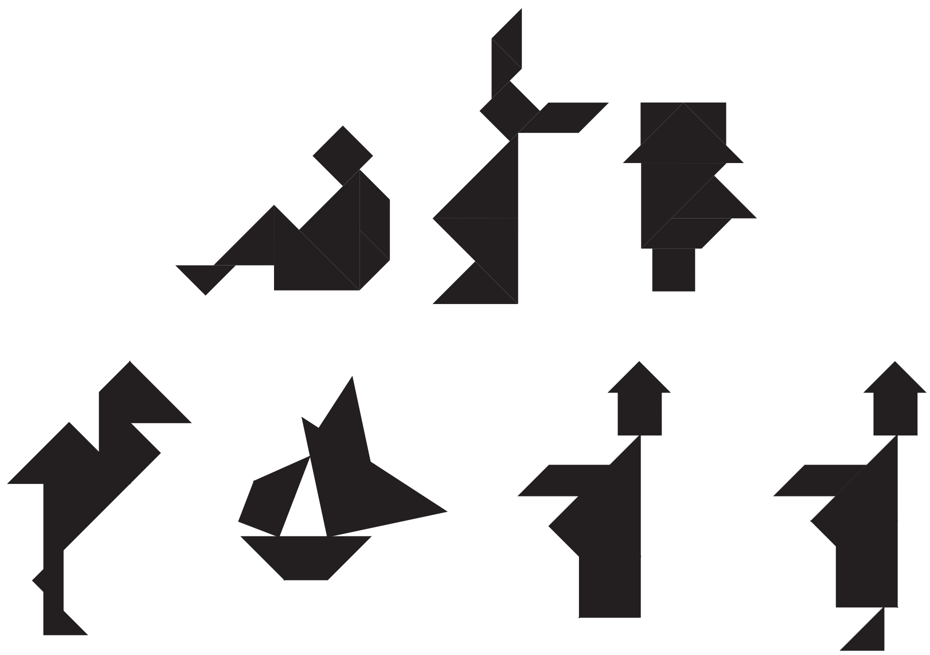
See page 173 for teacher notes to this section.

LAB 2.1
Meet the Tangrams

Name(s) ____________________________

■ **Equipment:** Tangrams

1. How many tangrams are there per set? __________ Make sure you have a complete set.
2. The tangram figures below appeared in *Amusements in Mathematics,* a 1917 book by British puzzlesmith Henry Dudeney. Try making some of them. Record the ones you made.



3. What interesting figures of your own can you make by combining all seven tangrams? Sketch them below and give them names.

4. What are the geometric names for each of the tangram shapes?

5. Trace each tangram shape. Flip it over. Does it still fit in the outline you traced?

6. What are the measures of the angles for each tangram shape? Find the angle measures without using a protractor, and write them down in the traced figures from Problem 5.

7. Write instructions you would give someone for making an accurate set of tangram pieces.

Discussion

A. How are the five tangram triangles related to one another?

B. How are the tangram triangles related to the other tangram shapes?

C. For each tangram, how much turning (how many degrees, or what fraction of a whole circle) do you have to do before it fits again in its traced outline?

D. Note that the last two figures shown on the previous page appear to be almost identical, except that one of them has a foot. Each of them was made with the whole set of seven tangrams. Where did the second man get his foot?

LAB 2.2
Tangram Measurements

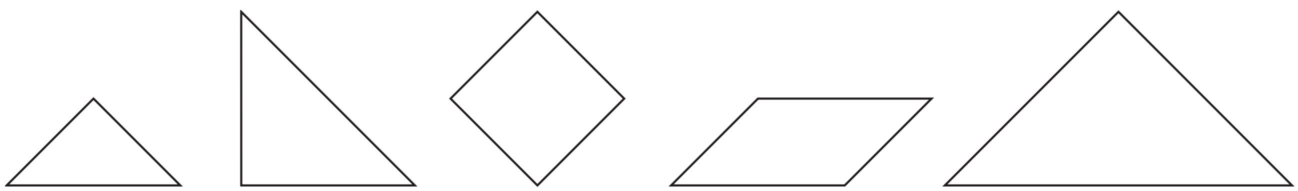
Name(s) ____________________________

- **Equipment:** Tangrams

Definition: The *hypotenuse* of a right triangle is the side opposite the right angle. The other two sides are the *legs*.

Fact: The hypotenuse of the small tangram triangle is exactly 2 inches in length.

1. Use that fact and logic to find as many of the other tangram side lengths as you can, as well as the area of each tangram piece. (Do not measure with a ruler!) If you get stuck, go on to the next piece, then come back to the ones you didn't get and try again. The rest of the activity will help you find measures you still can't get. Enter the length measurements along the sides of the reduced figures below, and write the areas inside the figures.



The rest of this lab will guide you through questions that should help you find any answers you did not find when working on Problem 1.

2. Cover the medium triangle with the two small triangles, then answer the following questions:
   - **a.** How long is one leg of the medium tangram triangle? __________
   - **b.** What is the area of the medium tangram triangle? __________
   - **c.** What is the area of each small tangram triangle? __________
   - **d.** What is the area of the tangram square? __________

Reminder: 49 is the square of 7, and 7 is the square root of 49.

3. Be careful when answering the following questions.
   - **a.** If the side of a square is 9 units long, what is the square's area? __________
   - **b.** If a square has an area of 9 square units, how long is one of its sides? __________
   - **c.** If the side of a square is 5 units long, what is the square's area? __________
   - **d.** If a square has an area of 5 square units, how long is one of its sides? __________

4. Generalize:

a. If the side of a square is *s* units long, what is the square's area? __________

b. If a square has an area of *A* square units, how long is one of its sides? __________

5. How long is the side of the tangram square? __________

6. Finish Problem 1.

7. Write an illustrated paragraph to explain how to obtain the measurements of all the tangram pieces from the hypotenuse of the small tangram triangle.

Discussion

A. In each of the tangram triangles, what is the ratio of the hypotenuse to the leg?

B. Each tangram triangle is a scaled version of each other tangram triangle. What is the *scaling factor* from the small to the medium? From the medium to the large? From the small to the large? (The scaling factor is the ratio of corresponding sides, *not* the ratio of areas.)

C. Make a square using the two large tangram triangles. What is its area? Express the length of its side in two different ways.

LAB 2.3

Name(s) ____________________

Tangram Polygons

- **Equipment:** Tangrams

- Make geometric figures, using any number of tangram pieces from one set.
- Keep track of your figures by checking boxes in the chart below.
- On the left side of the chart, add the names of other geometric figures you make, then check the box for the number of pieces you used in making them.

	How many pieces you used						
	1	**2**	**3**	**4**	**5**	**6**	**7**
Triangle							
Square							
Parallelogram							

Discussion

A. Which of the puzzles cannot be solved? For example, a one-piece, nonsquare rectangle is impossible. Are there others? Mark those puzzles with an X.

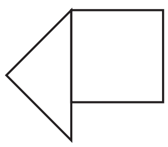
B. Explain why a six-piece square is impossible.

LAB 2.4
Symmetric Polygons

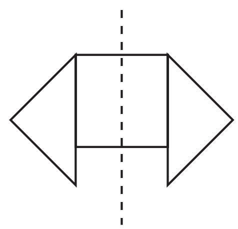
Name(s) ______________________

■ **Equipment:** Tangrams

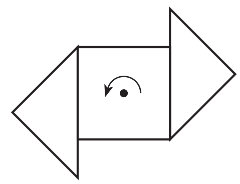
This tangram polygon is not symmetric:



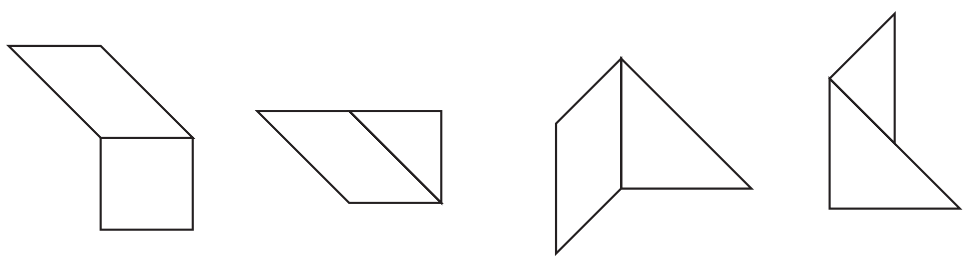
By adding a small tangram triangle, it can be made mirror symmetric:



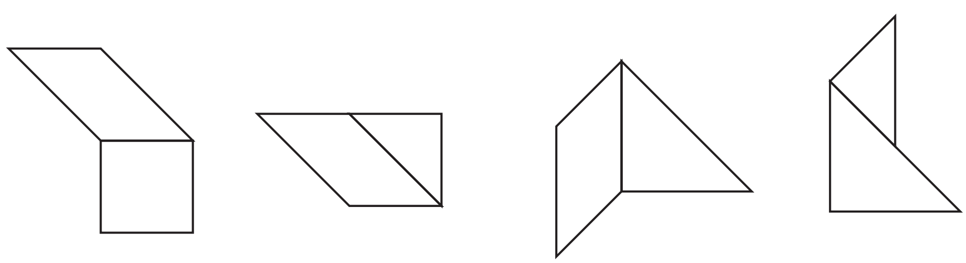
. . . or rotationally symmetric:



1. Add one or more tangram pieces to each tangram pair below to make mirror-symmetric polygons. Sketch your solutions on the figures.



2. Repeat Problem 1, making rotationally symmetric polygons.



3. Create your own symmetric tangram polygons. Sketch them on the back of this sheet.

Discussion

A. In Problems 1 and 2, is it more elegant to create symmetric figures by adding as few additional pieces as possible, or as many additional pieces as possible?

B. Create your own puzzles in the style of Problems 1 and 2.

C. Is it possible for a tangram polygon to exhibit both mirror *and* rotational symmetry?

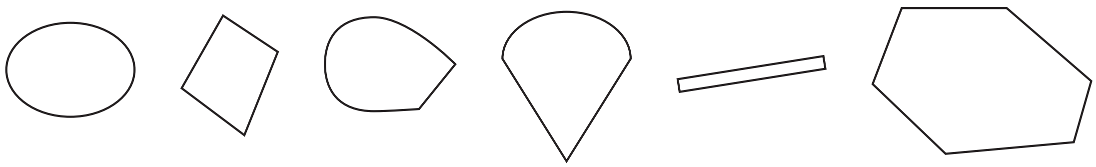
LAB 2.5

Name(s) ____________________________

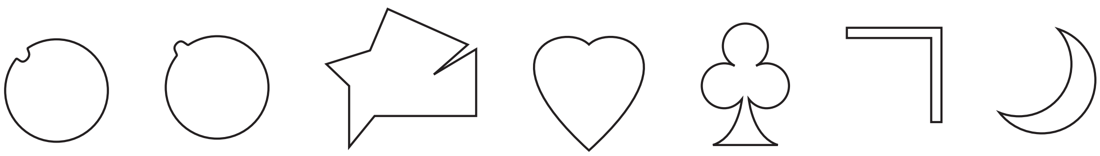
Convex Polygons

■ **Equipment:** Tangrams

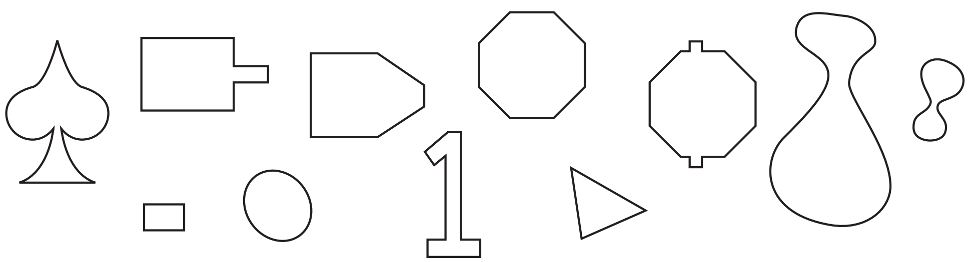
The figures below are convex.



The figures below are *not* convex.



1. Circle the convex figures below.



2. Draw a convex figure on the back of this sheet.
3. Draw a figure that is not convex on the back of this sheet.
4. Write down a definition of *convex* in your own words.

5. Find convex figures that can be made with tangram pieces. Sketch your solutions on the back of this sheet. (If you want an extra challenge, see how many seven-piece convex tangram figures you can find.)

Discussion

A. In your opinion, what makes a good tangram puzzle?

B. What makes a tangram puzzle easy or difficult?