Taxicab Geometry

If you can travel only horizontally or vertically (like a taxicab in a city where all streets run North-South and East-West), the distance you have to travel to get from the origin to the point (2, 3) is 5. This is called the taxicab distance between (0, 0) and (2, 3). If, on the other hand, you can go from the origin to (2, 3) in a straight line, the distance you travel is called the Euclidean distance, or just the distance. Taxicab distance can be measured between any two points, whether on a street or not. For example, the taxicab distance from (1.2, 3.4) to (9.9, 9.9) is the sum of 8.7 (the horizontal component) and 6.5 (the vertical component), for a total of 15.2.

Distance to One Point

1. What is the taxicab distance from (2, 3) to the following points?

   a. (7, 9) b. (–3,8) c. (2, –1)

   d. (6, 5.4) e. (–1.24, 3) f. (–1.24, 5.4)

2. Find all the points that are at a taxicab distance 5 from (5, 5).
   a. Show your findings on graph or dot paper.
   b. Explain why the set of those points is called a *taxi-circle.*
   c. The number π is the ratio of the perimeter of a circle to its diameter. In Euclidean geometry, $\pi = 3.14159\ldots$ Find the value of *taxi-*π.

Distance to Two Points

Answer #3-8 for each of these cases:

a. A(0, 0) and B(6, 0) b. A(0, 0) and B(4, 2) c. A(0, 0) and B(3, 3)

d. A(0, 0) and B(1, 5)

3. **Equidistance.** Find the set of points that are equidistant from A and B.
4. **Taxi-Ellipses.** Find the set of points P such that PA + PB = 6
5. Find the set of points P such that PA + PB = 10
6. **Taxi-Hyperbolas.** Find the set of points P such that $|PA - PB| = 4$
7. Find the set of points P such that $|PA - PB| = 6$
8. Find the set of points P such that PA/PB = 2 (Careful! The last case is particularly tricky.)

Distance from a Point to a Line

9. How would you define taxicab distance from a point to a line? Consider different cases, depending on the line's slope, *m*:
   a. $m = 0$ (the line is *horizontal*)
   b. $0 < |m| < 1$ (the line is *gentle*)
   c. $|m| = 1$ (the line makes a *45° angle* with the axes)
   d. $|m| > 1$ (the line is *steep*)
   e. *m* is undefined (the line is *vertical*)

10. **Taxi-Parabolas.** Given a line *d* and a point F not on *d*, find the set of points that are equidistant from *d* and F. Consider each of the cases listed in Problem 7.

Discussion / Extensions

A. Given two points A and B, find the set of points P such that PA + PB is minimal. Investigate the same question for three or even more points.

B. Given two points A and B as in **Distance to Two Points**, in what cases do we have no taxi-ellipse? no taxi-hyperbola?

C. Which is usually greater, taxicab or Euclidean distance? Can they be equal? If so, in what cases?

D. Find a formula for the taxicab distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. (**Hint**: Start by figuring out a formula for the case where the points are on a common horizontal or vertical line. The formula should work whether P_1 or P_2 is named first.)

E. In Euclidean geometry, for three points A, B, and C, we always have AB + BC ≥ AC.This is called the *triangle inequality*. Does it work in taxicab geometry? If so, in what cases do we have equality?

F. Explain how to find the center of a taxi-circle that goes through two points A and B and, once you have the center, how to sketch the circle. Give examples based on the cases listed in Problem 3.

G. In Euclidean geometry, three noncollinear points determine a unique circle, while three collinear points determine no circle. In taxicab geometry, the situation is somewhat more complicated. Explore different cases, and try to find out when three points determine no circle, one circle, or more than one circle.