## Congruence and Similarity Teacher Notes

Starting in $8^{\text {th }}$ grade, congruence and similarity are defined based on geometric transformations.
In these lessons, the figures are not labeled as to which one is a pre-image because it can work in either direction.

## 1. Congruent Figures

This follows up on a similar activity in the Isometries packet (Lesson 4: Isometries Puzzles.)
Note that the lesson starts with a built-in review, so you need not conduct a review at the board to launch the activity. However, locating the center for \#1c is difficult. It is OK for students to skip that part if they have trouble with it.
\#5-6 connects with the traditional definition of congruence for polygons: equal sides and equal angles.

## 2. Similar Figures

Again, the lesson starts with a built-in review, so you need not conduct a review at the board to launch the activity. Note that \#1c can serve as a hint to \#1b!
\#1d can serve as an opportunity to discuss the fact that when you switch to the other polygon being the pre-image, the new scaling factor is the reciprocal of the old one.

In \#2-5, make sure the students specify whether the rotation is clockwise or counterclockwise.

## 3. Similarity

\#1 is a problem in the style of the previous lesson.
\#2 connects with the traditional definition of similarity for polygons: proportional sides and equal angles.
\#3 is intended as a wrap-up of the concept of similarity. You may ask the students to discuss the questions with their neighbors in preparation for a class-wide discussion.

## 4. A Special Relationship

If students have trouble with \#3, you might first show them examples from the Dilations unit as a reminder of what needs to be done to turn the shape upside down (a negative scaling factor.) Then, you can ask how the ratio of the sides can be used to figure out what negative number would do the job.
\#5 offers an opportunity to summarize many key ideas of the Isometries and Dilations units, as well as this unit.

## 1. Congruent Figures

Two figures are considered congruent if you can get one from the other by a sequence of isometries.

1. In these examples, the figures are congruent. You can get from one to the other with a single basic isometry. Name the isometry in each case. If it is a reflection, draw the line of reflection. If it is a rotation, draw the center of rotation. If it is a translation, draw the vector.


Sometimes, you need to use more than one basic isometry to get from one figure to the other. The example on the left shows two polygons:


We can tell they are congruent because we can get from one to the other by using isometries. There are many ways to do it. The figure on the right shows one possible way. If you start with the top left polygon, you can use a reflection, then a translation to get to the bottom right polygon.
2. a. Draw the reflection line and the translation vector on the figure.
b. You could also show the polygons are congruent by starting on the bottom right polygon. List the isometries you would use to get the top left polygon. Use the figure on the left side above to draw the intermediate polygon, and show any needed details, such as lines, points or vectors.
3. In the figure below, show that the two polygons are congruent by explaining in detail at least one way to get from one to the other using isometries. Add any needed lines, points or vectors to the figure to help your explanation, as well as the intermediate polygon(s).

4. Repeat \#3 for these two polygons:

5. When two polygons are congruent, what can you say about a. their sides?
b. their angles?
6. If two circles are congruent, what can you say about their radii?

## 2. Similar Figures

Two figures are considered similar if you can get one from the other by a sequence of isometries and dilations. Here are several examples.

These two figures are connected by a dilation, therefore they are similar:


1. a. Where is the center of dilation?
b. What is the scaling factor if the smaller polygon is the pre-image?
c. To find the scaling factor in parts $b$ and $c$, did you use the distances from the center of dilation, the side lengths of the figures, or something else?
d. What is the scaling factor if the larger polygon is the pre-image? (Hint: this can be answered without any more measurements!)

The two polygons in the illustration on the left, below, are connected by an isometry and a dilation, therefore they are similar.

In fact, they could be connected by a rotation then a dilation, or a dilation then a rotation:


In the illustration on the right, an intermediate step is shown. Refer to this illustration for problems 2 and 3 .
2. If the smaller polygon is the pre-image:
a. Which happened first, the rotation or the dilation?
b. Which point $(\mathrm{O}$ or P$)$ is the center of rotation? What is the angle?
c. Which point is the center of dilation? What is the scaling factor?
3. If the larger polygon is the pre-image:
a. Which happened first, the rotation or the dilation?
b. Which point is the center of dilation? What is the scaling factor?
c. Which point is the center of rotation? What is the angle?


The illustration on the right, above, shows a different intermediate step. Refer to this illustration for problems 4 and 5.
4. If the smaller polygon is the pre-image:
a. Which happened first, the rotation or the dilation?
b. Which point is the center of rotation? What is the angle?
c. Which point is the center of dilation? What is the scaling factor?
5. If the larger polygon is the pre-image:
a. Which happened first, the rotation or the dilation?
b. Which point is the center of rotation? What is the angle?
c. Which point is the center of dilation? What is the scaling factor?

## 3. Similarity

1. Show that the two polygons below are similar by finding a sequence of isometries and dilations to get from one to the other. You can choose which polygon is the pre-image. (Hint: it can be done in two or more moves.) Explain your strategy. Add any needed lines, points or vectors to the figure to help your explanation, as well as the intermediate polygon(s).

2. If two polygons are similar, what can you say about
a. their sides?
b. their angles?

## 3. Discussion:

a. Explain why any two squares are similar.
b. Give examples of rectangles that are similar.
c. Give examples of rectangles that are not similar.
d. Explain why any two circles are similar.

## 4. A Special Relationship

The polygons in this figure have a special relationship.


1. Use the figure above to show how you can get from one to the other with a rotation. Mark the center. What is the angle?

2. Use the figure above to show how you can get from one to the other with two reflections. Draw the lines of reflection.

3. Use the figure above to show how you can get from one to the other with a dilation. Show the center. (Hint: the center of dilation will be the same as the center of rotation in \#1.) What is the scaling factor? (Hint: what sort of scaling factor turns a figure upside down?)
4. Are the two polygons congruent, similar, both, or neither? Justify your answer.
5. Discussion. When two figures are similar with a ratio of 1 or -1 , they are also congruent. Explain.
