Three Quadratic Forms

*Only use an electronic grapher to check your answers!*

**Factored Form**

1. For the function $y = 2 (x + 3) (x - 4)$:
   a. What are the roots (the x-intercepts)? (Hint: what is the value of $y$ at the x-intercepts?)
   b. What is the y-intercept?
   c. What is the x-coordinate of the vertex? (Hint: it’s half way between the roots)
   d. What is the y-coordinate of the vertex?

The following questions are about *factored form*: $y = a (x - p)(x - q)$

2. How do we know $p$ and $q$ are the roots?

3. Find, in terms of $a$, $p$, and $q$:
   a. The y-intercept.
   b. The x-coordinate of the vertex.
   c. **Challenge**: The y-coordinate of the vertex.

**From Factored Form to Standard Form**

$y = ax^2 + bx + c$ is called *standard form*. Some of the information that is obvious in factored form is not as visible in standard form.

4. Take the equation $y = a (x - p)(x - q)$, and distribute, so as to write it in standard form. [Hint: this is a two-step process. One way to do it is to first multiply $a(x - p)$. Then multiply the product by $(x - q)$.]

5. Write formulas for $b$ and $c$ in terms of $a$, $p$, and $q$. The trickiest part is to figure out $b$ — it requires factoring the $x$.

6. Find, in terms of $a$, $b$, and $c$:
   a. The y-intercept.
   b. The sum of the roots.
   c. The product of the roots.
   d. The x-coordinate of the vertex.
   e. **Challenge**: The y-coordinate of the vertex.

Note that the x-coordinate of the vertex does not depend on $c$. It follows that the formula still works even if the function has no roots: by changing $c$, we can change it to a function that does have roots without affecting the x-coordinate of the vertex.
Vertex Form

7. Consider the expression $2 (x - 3)^2 + 4$
   a. What is the smallest value it can possibly be? Explain, without referring to a graph.
   b. For what value of $x$ does it reach this smallest value? Explain, without referring to a graph.

8. Consider the expression $a (x - h)^2 + v$, with $a > 0$
   a. What is the smallest value it can possibly be? Explain.
   b. For what value of $x$ does it reach this smallest value? Explain.
   c. What happens if $a < 0$?

   It follows that $(h, v)$ are the coordinates of the vertex of the parabola with equation $y = a (x - h)^2 + v$. This is vertex form.

From Vertex Form to Standard Form

Some of the information that is obvious in vertex form is not as visible in standard form.

9. Take the equation $y = a (x - h)^2 + v$, and distribute, so as to write it in standard form.

10. Write formulas for $b$ and $c$ in terms of $a$, $h$, and $v$.

11. Find $h$ in terms of $a$ and $b$.

12. Challenge: Find $v$ in terms of $a$, $b$, and $c$.

Are There Roots?

The y-coordinate of the vertex, which you might have found in #6e or 12 is $v = \frac{-b^2 + 4ac}{4a}$.

If you got a different answer, figure out whether you made a mistake, or whether your solution was equivalent to this one.

We will use $a$ and $v$ to determine if a quadratic function has roots.

13. Since $a \neq 0$, there are two cases:
   a. If $a > 0$, we have a “smile” parabola. In this case, the parabola intersects the x-axis if ________
   b. If $a < 0$, we have a “frown” parabola. In this case, the parabola intersects the x-axis if ________

14. Challenge: Find an inequality involving only $a$, $b$, and $c$ that one can use to figure out whether the parabola intersects the x-axis. (Hint: It can be deduced from the two cases above and the formula for $v$.)
Make These Parabolas (TI-83/84)

WINDOW
Xmin=-9.4
Xmax=9.4
Xscl=1
Ymin=-6.2
Ymax=6.2
Yscl=1
Xres=1
Make These Parabolas (TI-89)

To split the screen: **MODE** *F2* then **ENTER**

To move between screens, **2nd** **APPS**

After you have set the window as indicated above, go to the Y= screen on the right of your split screen. Enter functions in the Y= screen, then press **2nd** **APPS** to see their graphs.

Make these parabolas: